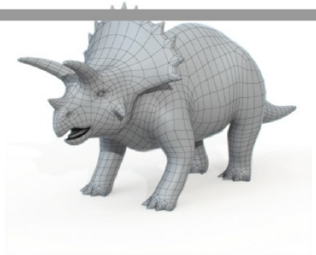
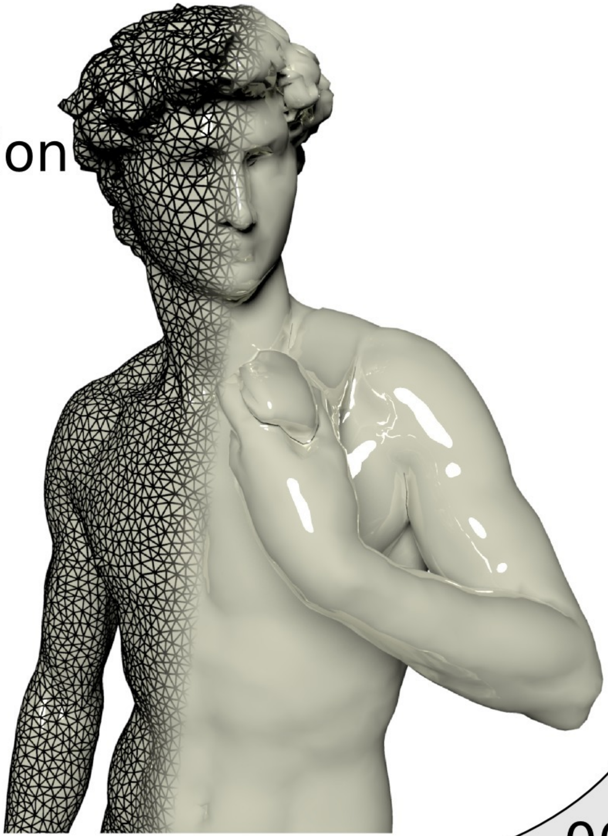


3D Modeling Methods

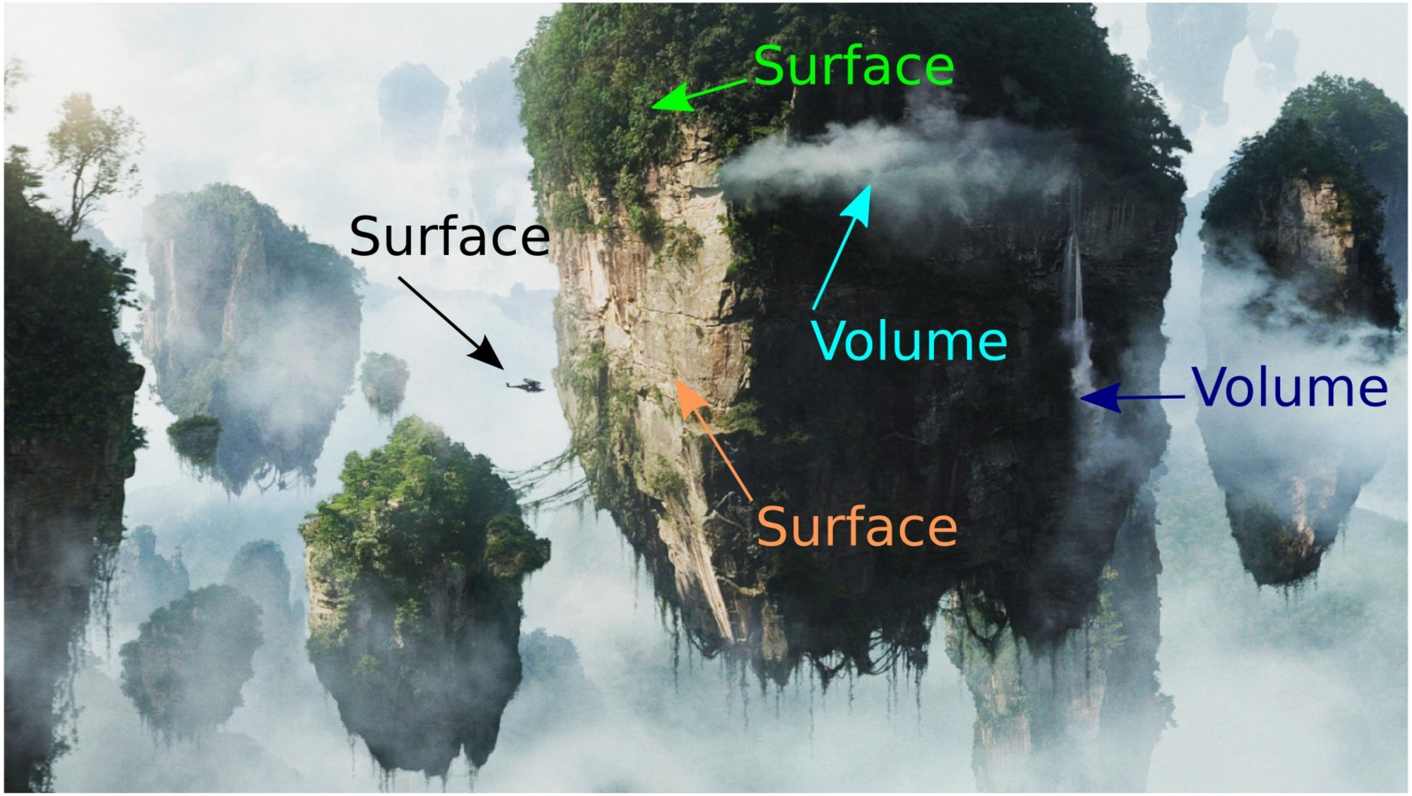
Modeling methods: Objective

- Understand how to model a 3D object
- Which model is the most suitable for a given application



How do we model a 3D object

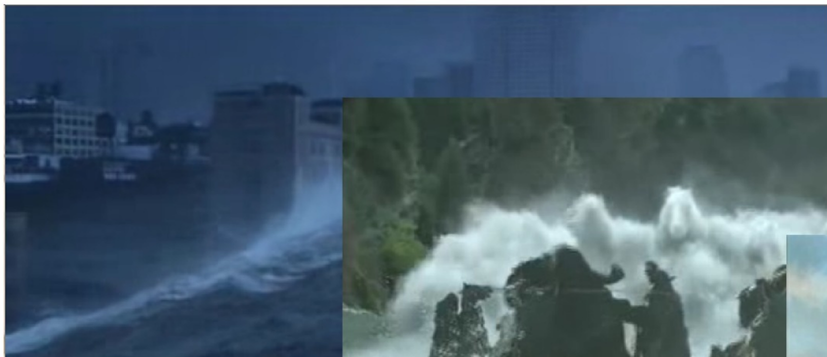
1. Surface modeling VS Volume modeling
2. How do we encode it on a computer



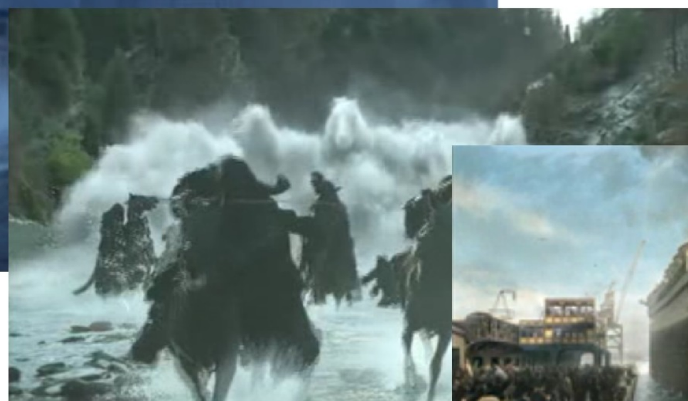
[Avatar]

3D model realism

Which are the virtual objects ?



[Day After Tomorrow]

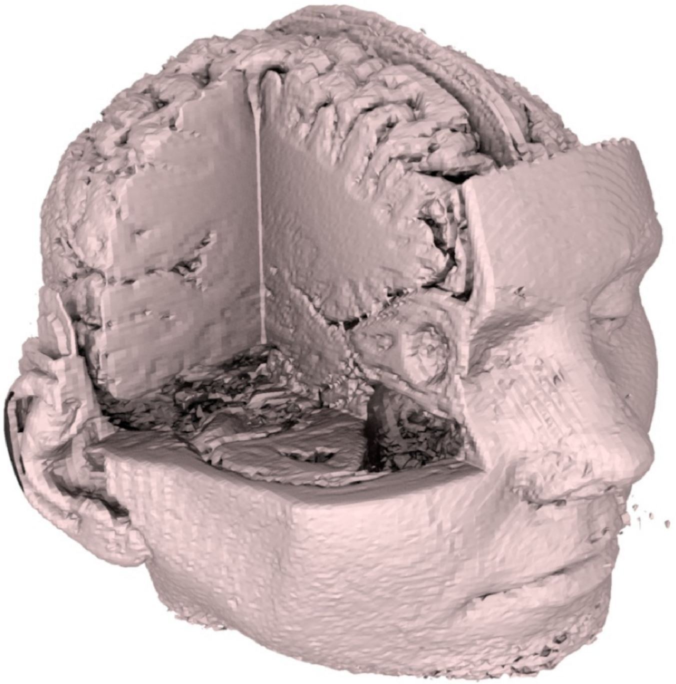


[Lord of the Rings]

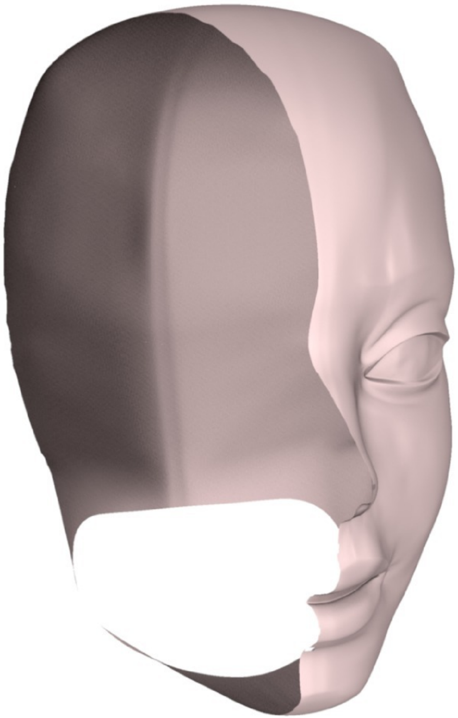
[Titanic]

Modeling a 3D object

Volume modeling



Surface modeling



Representing a Surface

Explicit

- BRep: *Boundary Representation*
  - Mesh
  - Parametric
  - Subdivision
- CSG: *Constructive Solid Geometry*

Implicit

- Voxels
- Parametric
  - Skeleton
  - Analytic
- Point-Sets
  - MLS
  - Surfels

Fractals

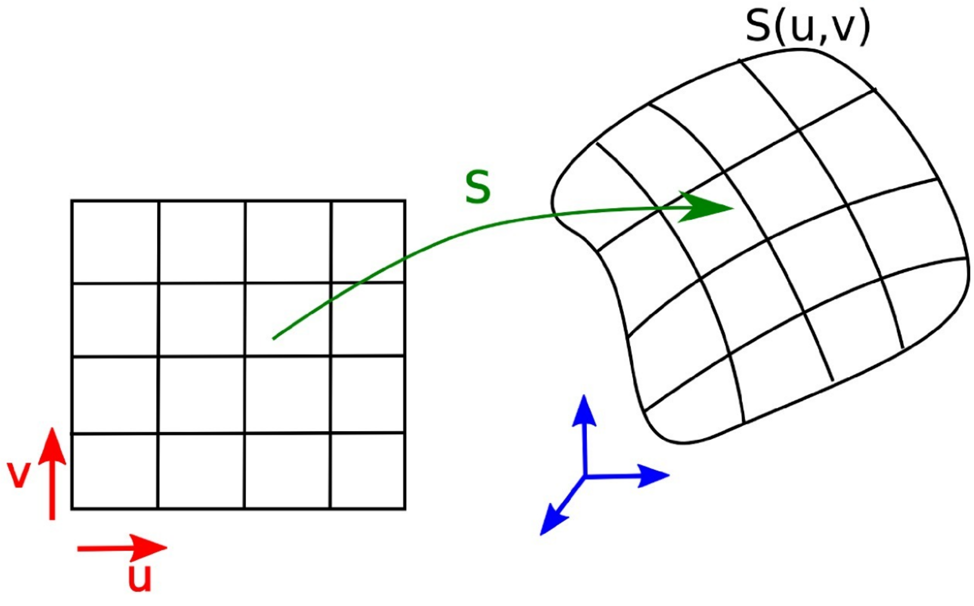
Explicit representation

Mesh
Parametric
Subdivision

Explicit representation

Explicit representation ~ parametric

$$S : \begin{cases} \mathcal{D} \subset \mathbb{R}^2 & \to \mathbb{R}^3 \\ (u,v) & \mapsto S(u,v) = (S_x(u,v), S_y(u,v), S_z(u,v)) \end{cases}$$



$$S = \text{mapping} \neq \underbrace{\text{Surface } \Gamma}_{\text{trace of } S \text{ in } \mathbb{R}^3}$$

Triangular Mesh

Triangular mesh = Simplest BRep

When S is not known analytically

We compute a discrete local approximation

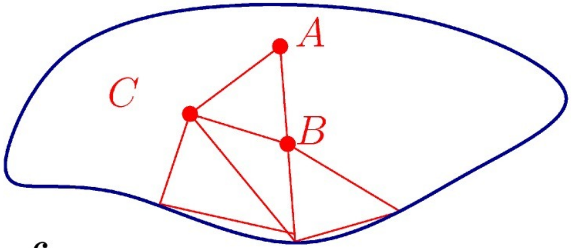
$$S = \bigcup_i S_i$$

Simplest mapping : Linear mapping

$$S_i : \begin{cases} \mathcal{D} \subset \mathbb{R}^2 & \to \mathbb{R}^3 \\ (u, v) & \mapsto S_i(u, v) = u\,\overrightarrow{AB} + v\,\overrightarrow{AC} + \overrightarrow{OA} \end{cases}$$

$$\mathcal{D} : (u, v) \in [0, 1]^2, 0 \leq u + v \leq 1$$

- S is globally $\mathcal{C}^0$
- S is never $\mathcal{C}^1$ (exc. plane)
- S can interpolate any discrete set of points on a surface



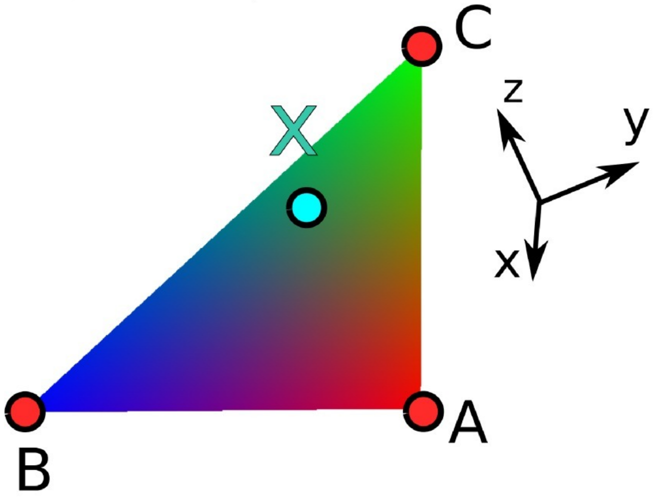
Barycentric Coordinates

$$(u, v) \mapsto S_i(u, v) = u\,\overrightarrow{AB} + v\,\overrightarrow{AC} + \overrightarrow{OA}$$

$$(\alpha, \beta, \gamma) \mapsto S'_i(\alpha, \beta, \gamma) = \alpha\,A + \beta\,B + \gamma\,C$$

$$\alpha + \beta + \gamma = 1$$

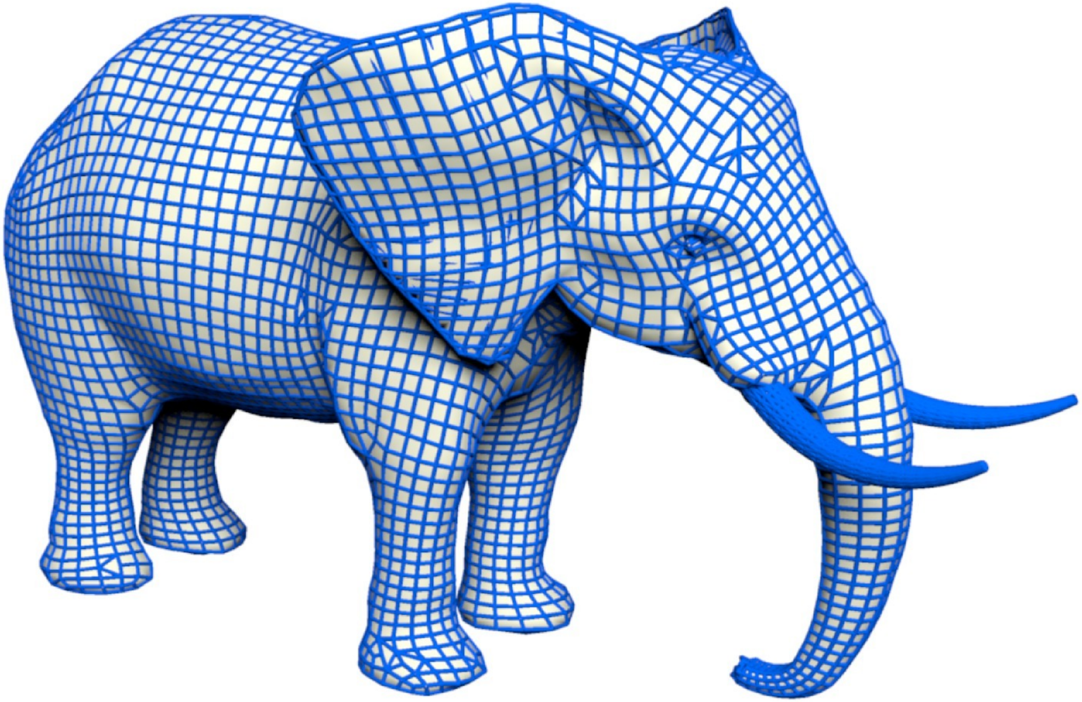
$$(\alpha, \beta, \gamma) \in [0, 1]^3$$



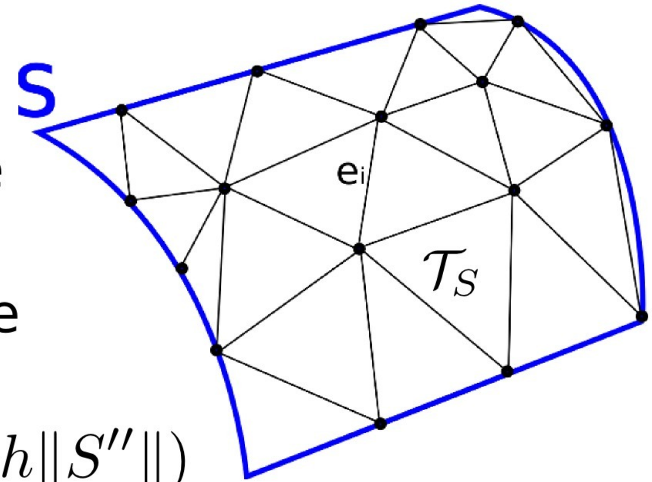
Mesh

Special case:
A mesh can have polygons made of N vertices (N>3)

A good polygon should have N coplanar vertices
(otherwise, need triangulation)



Mesh : Approximation



S : true smooth surface

$\mathcal{T}_S$: triangulated surface

$$\|S - \mathcal{T}_S\| = h\,|\kappa_{\max}|\ (\simeq h\|S''\|)$$

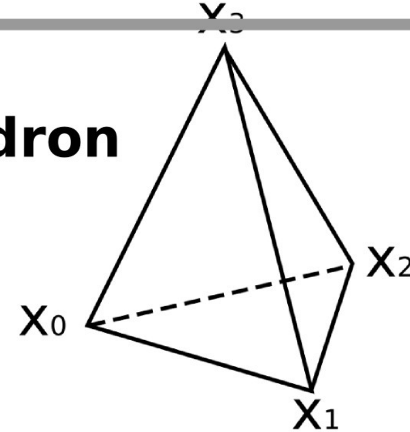
Linear approximation of a surface S (first order)

$$h = K\ \max_i e_i$$

$$\Rightarrow \|S - \mathcal{T}_S\| = \mathcal{O}\left(\max_i e_i\ |\kappa_{\max}|\right)$$

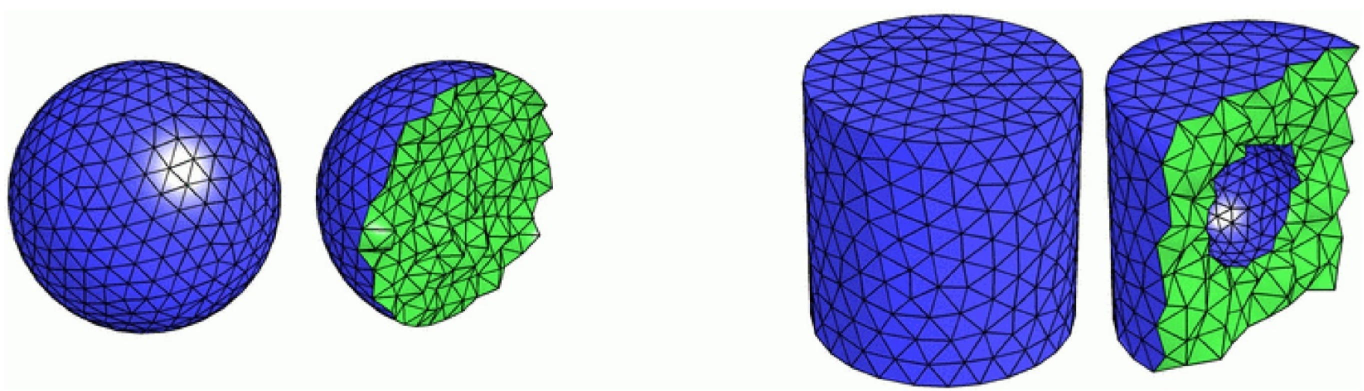
Mesh : Volume

Linear element in volume = **Tetrahedron**



$$\mathbf{x} = u(\mathbf{x}_1 - \mathbf{x}_0) + v(\mathbf{x}_2 - \mathbf{x}_0) + w(\mathbf{x}_3 - \mathbf{x}_0) + \mathbf{x}_0$$
$$0 < u + v + w < 1$$
$$(u, v, w) \in [0, 1]^3$$

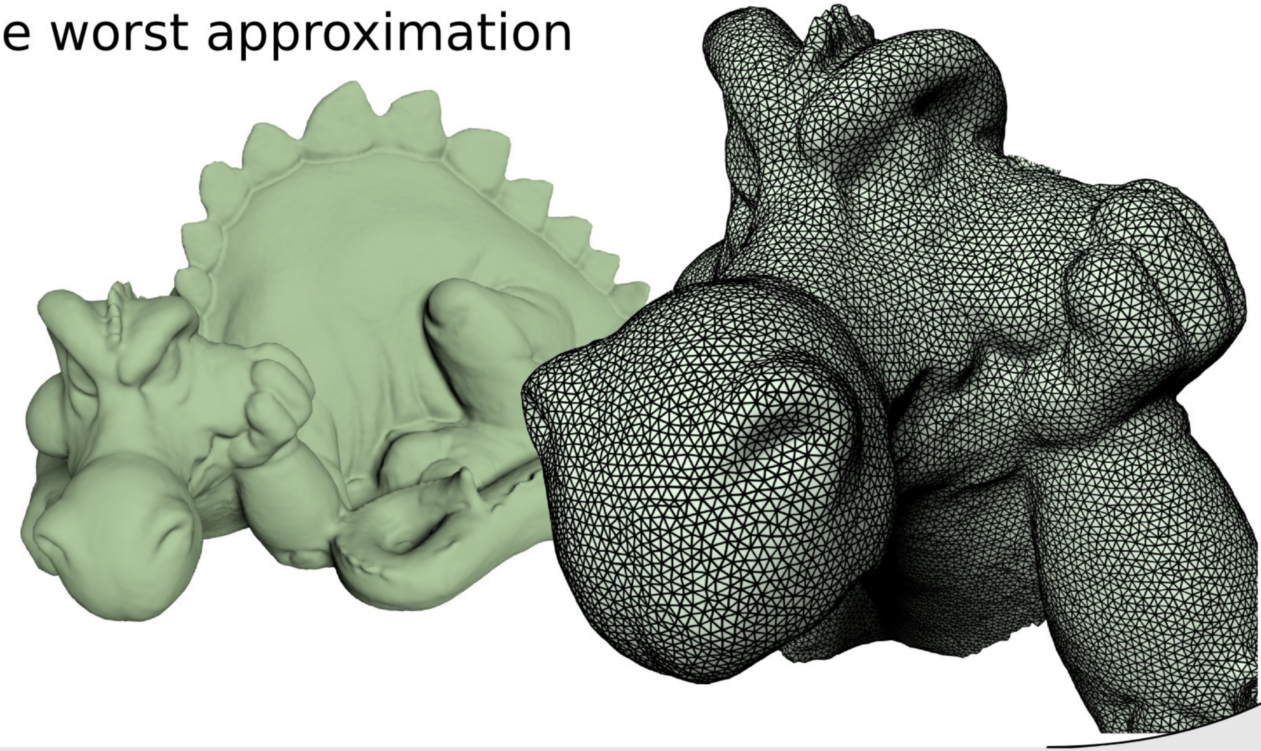
$$\mathbf{x} = \alpha\mathbf{x}_0 + \beta\mathbf{x}_1 + \gamma\mathbf{x}_2 + \delta\mathbf{x}_3$$
$$\alpha + \beta + \gamma + \delta = 1$$
$$(\alpha, \beta, \gamma, \delta) \in [0, 1]^3$$



Mesh, conclusion

+ The simplest
+ The most versatil
+ The most common
+ Fast dedicated rendering

- The worst approximation



Explicit representation

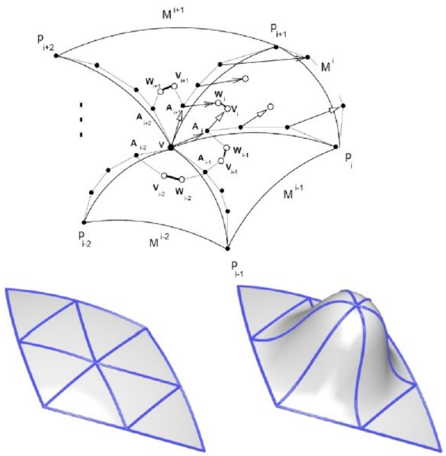
Mesh
Parametric
Subdivision

Parametric

- Order > 1

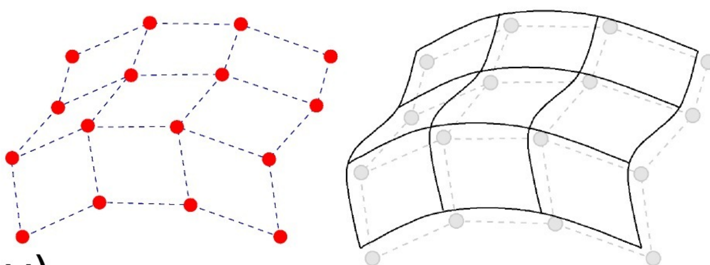
- Idea 1
  - Derivative data
    (smooth triangles)
  - Problem: Difficult to set-up

  => **Not so used**



[Yvart, Hahmann, 01-04]

- Idea 2
  - Add extra vertices
    (non triangular patches)
  - Problem: Patch structure
  - Common case:
    Rectangular patches (u,v)



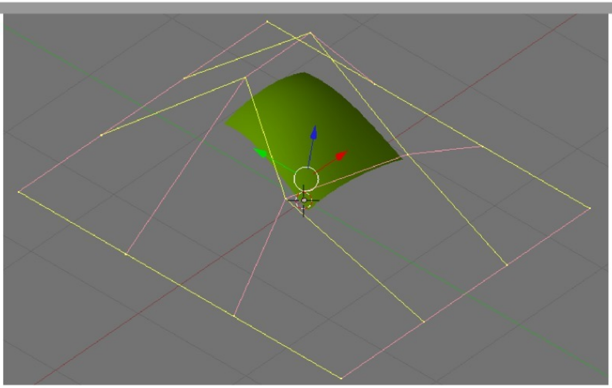
  => **Very common**

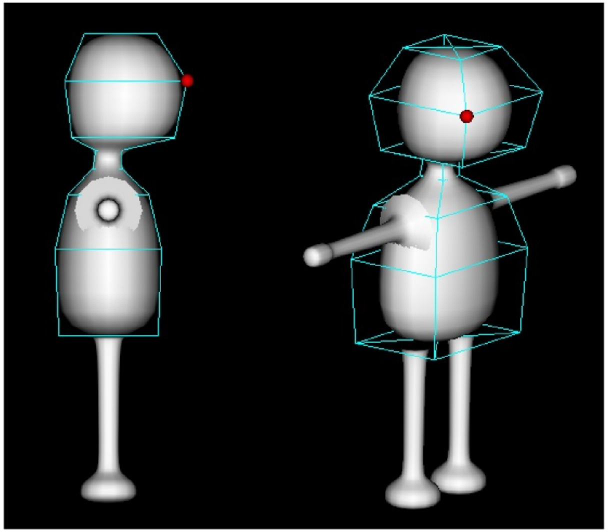
Parametric : Spline Patches

- Patch (4x4) : bicubic polynomial
- Parametric surface C^2
  (Continuous curvature)

$$S(u,v) = \sum_{i=0}^{3}\sum_{j=0}^{3} b_i(u)\, b_j(v)\, \mathrm{P}_{ij}$$

(tensorial product surface)





- Special case
  (uniform knot vector)

$$S(u,v) = (u^3 u^2 u\ 1)\, \mathrm{M}\, [P_{ij}]\, \mathrm{M}^T\, (v^3 v^2 v\ 1)^T$$

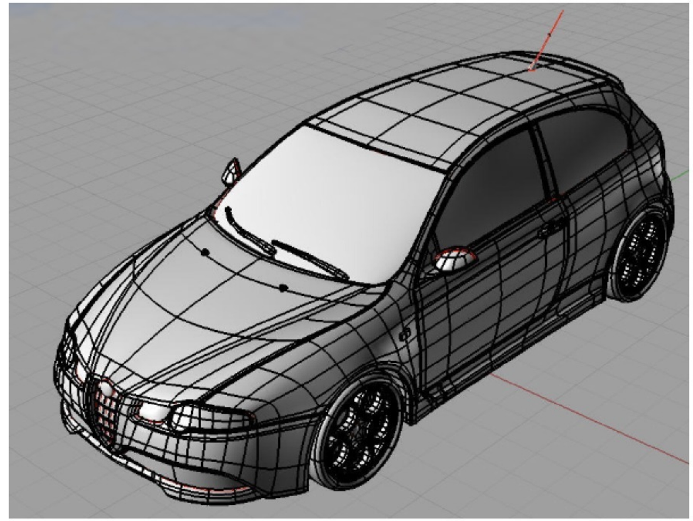
$$\mathrm{M} = \frac{1}{6}\begin{bmatrix} -1 & 3 & -3 & 1 \\ 3 & -6 & 3 & 0 \\ -3 & 0 & 3 & 0 \\ 1 & 4 & 1 & 0 \end{bmatrix}$$

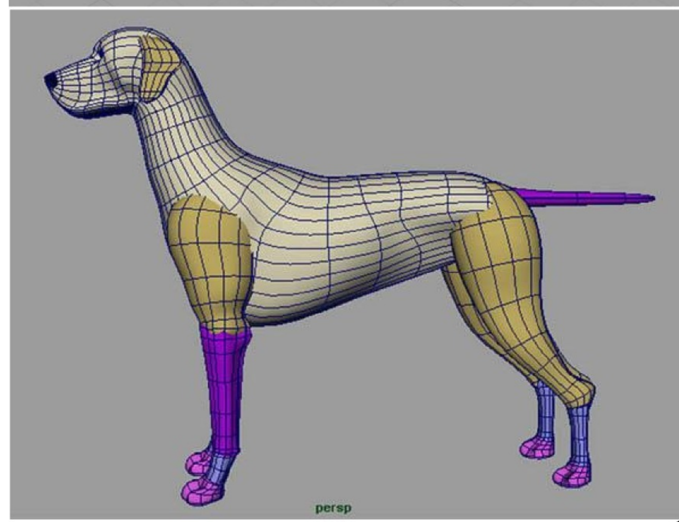
Parametric, Conclusion

+ Smooth Surface (CAD)

- Patch Structure

  Manual modeling
  Technical
  Junction



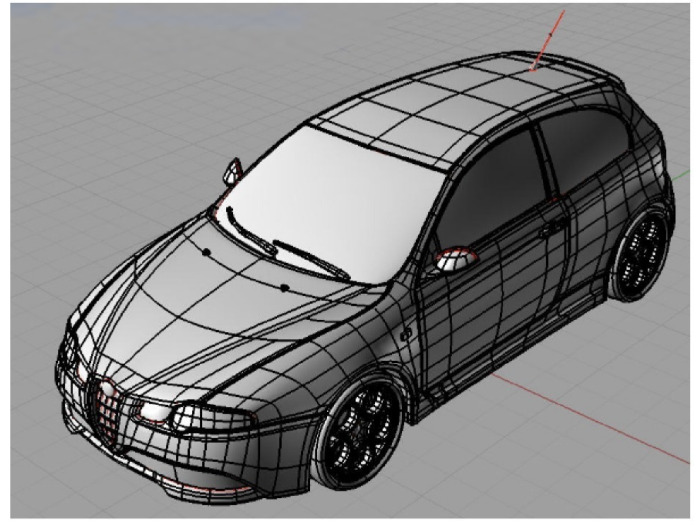


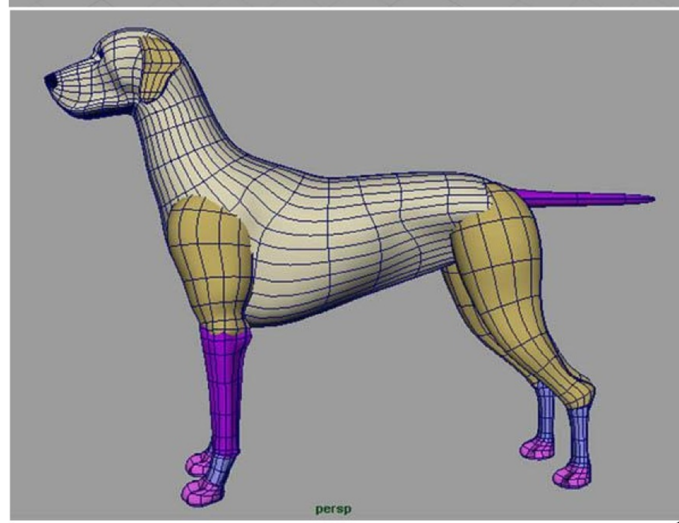
Parametric, Conclusion

+ Smooth Surface (CAD)

- Patch Structure

  Manual modeling
  Technical
  Junction





Explicit representation

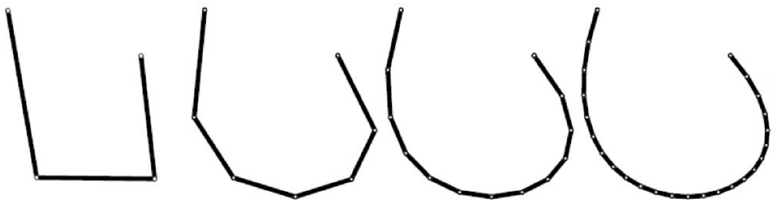
Mesh
Parametric
Subdivision

Subdivision surface

Answer the problem

+ Smooth surface
+ Local control
+ Arbitrary topology

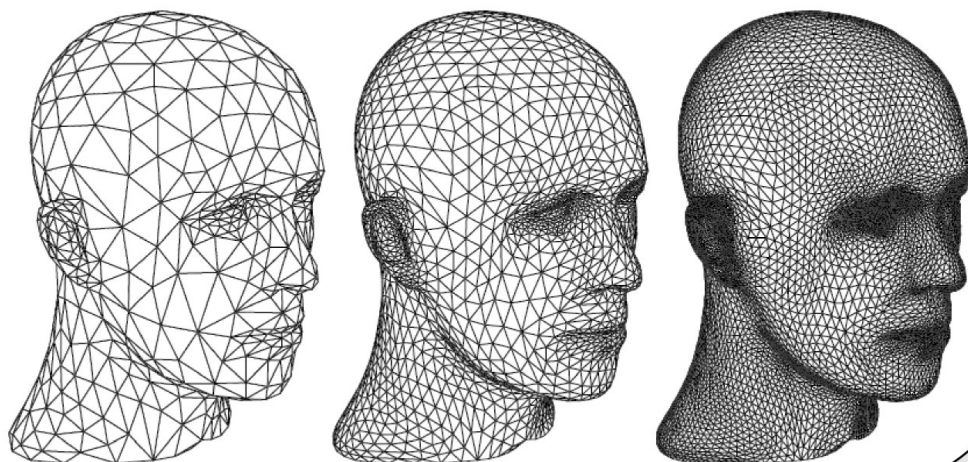
- Principle in 1D (subdivision curve)



$$\begin{pmatrix} x_n^{2k} \\ x_n^{2k+1} \end{pmatrix} = \begin{pmatrix} 3/4 & 1/4 \\ 1/4 & 3/4 \end{pmatrix} \begin{pmatrix} x_{n-1}^{k} \\ x_{n-1}^{k+1} \end{pmatrix}$$

- Principle in 2D

2D subdivision mask

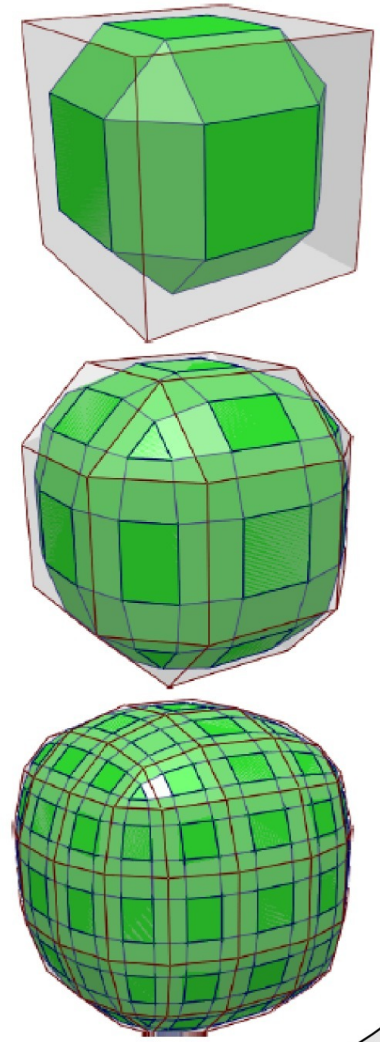


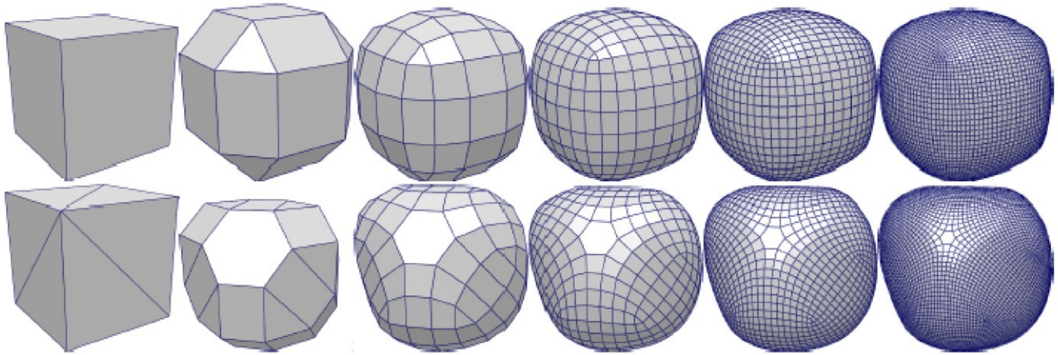
[Zorin, Schroeder, SIGGRAPH Course Notes 99]

Subdivision surface, construction

Step 0: Control polygon P0
Step 1: Subdivision P^0 -> P^1
...
Step i: Subdivision P^{i-1} -> P^i
...
Limit surface $S = \lim_{i \to +\infty} P^i$

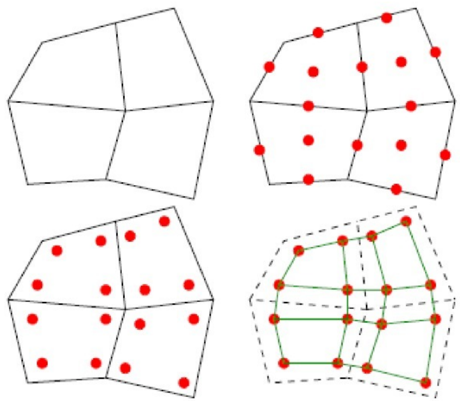
S can be C^2 almost everywhere



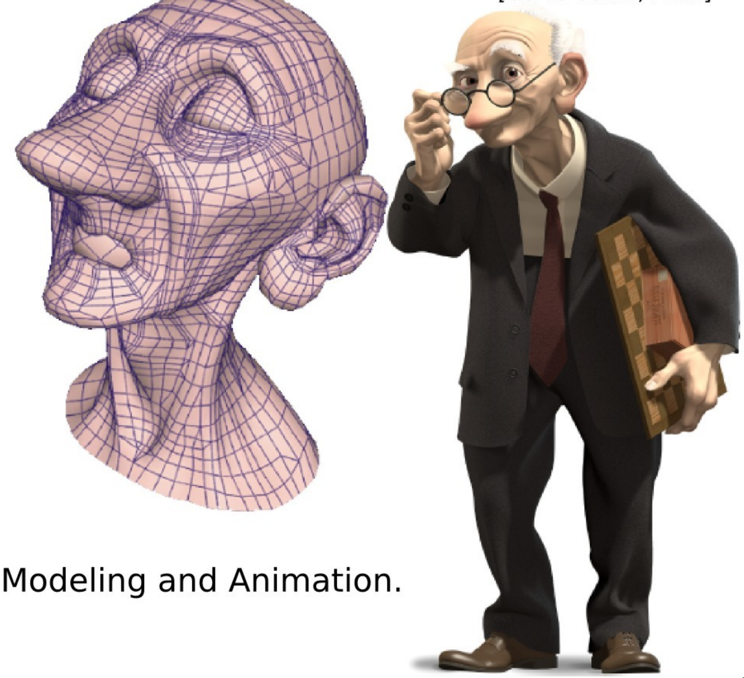


Subdivision scheme

Loop : triangles, C^2 alm. ev., approximation
Catmull Clark: quads, C^2 alm. ev., approximation
Doo-Sabin (corner cutting): quads, C^2 alm. ev., approximation
Butterfly: triangles, C^1 alm. ev., interpolation.
Kobbelt: triangles, C^2 alm. av., approximation



[Geri's Game, Pixar]

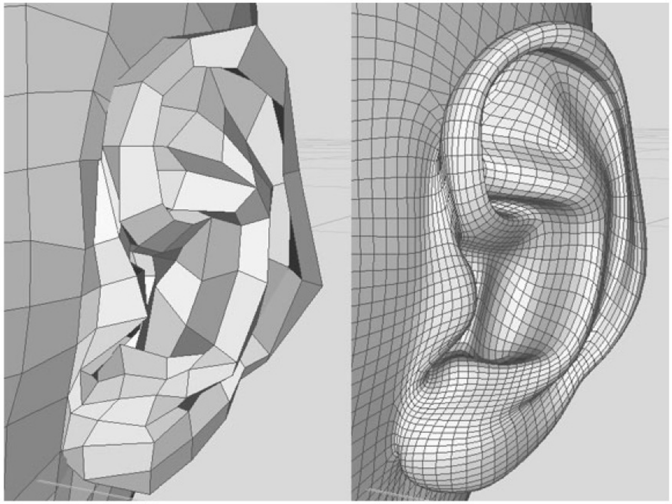


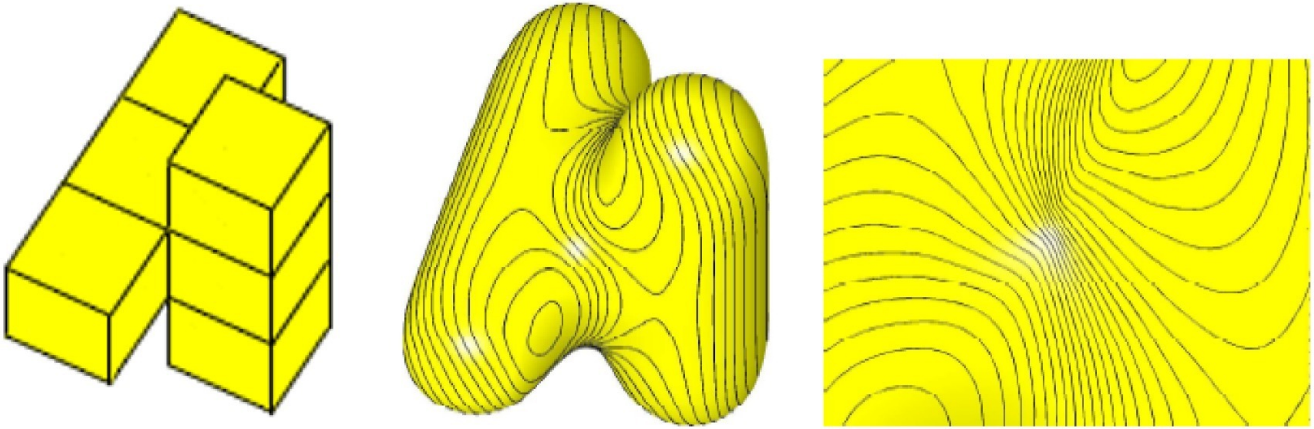
D. Zorin, P. Schroder. Subdivision for Modeling and Animation.
ACM SIGGRAPH Course Notes, 1999.

Subdivision surface, conclusion

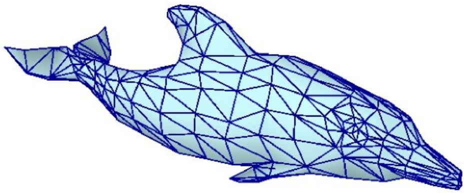
+ Arbitrary topology
+ Smooth surface

- Control of the final shape
- Extraordinary vertices





BRep: Comparison



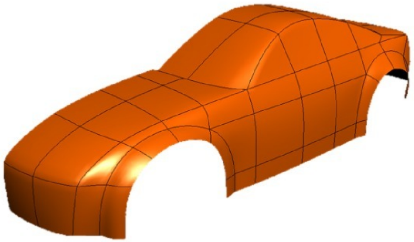
Mesh

+ Simple
+ Generic
+ Automatic generation

- Non differentiable
- Bad approximation
- Manipulation

Graphics, Computation.

Maya, 3DStudio, Blender



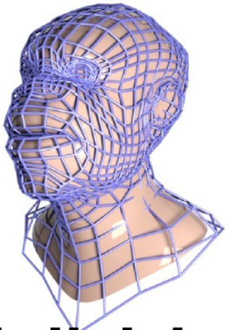
Parametric

+ Smoothness
+ Parametrization information

- Technical (math. model)
- Patch structure
- Manual generation

CAD, (Graphics)

Rhino, Catia, Autodesk



Subdivision

+ Smooth appearance

+/- No Patch, Extr. vertices

- No information on limit surface

Graphics, (CAD)

Explicit : CSG

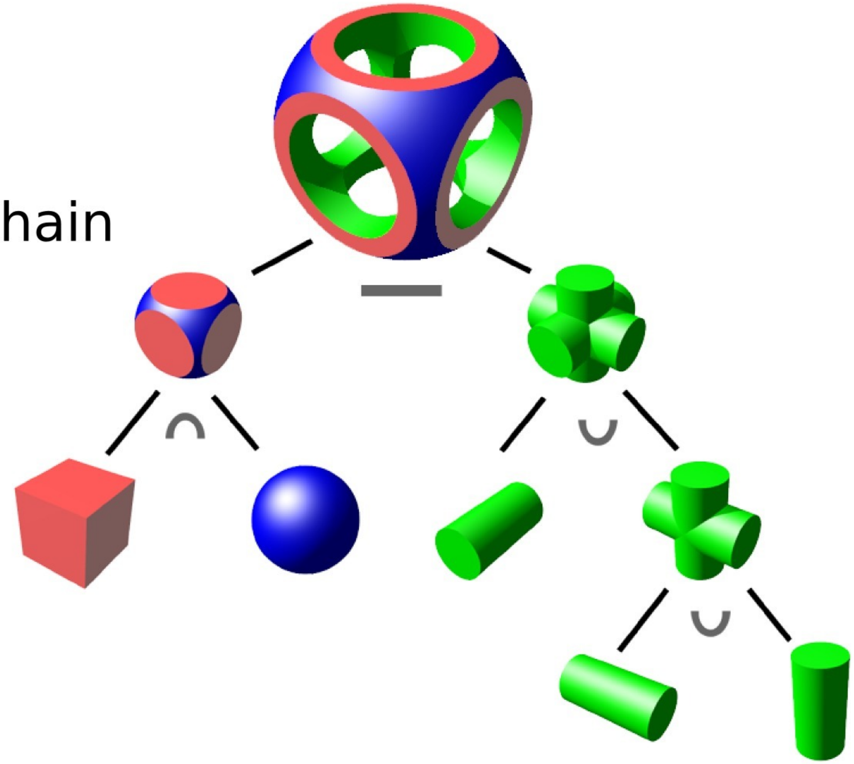
CSG: Constructive Solid Geometry

CSG = Assembling primitives using boolean operations

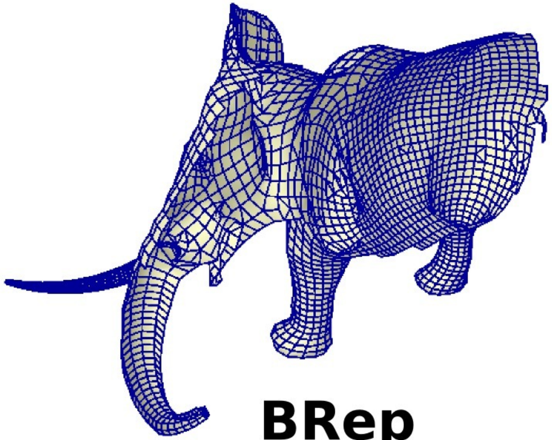
- Solid: Interior/Exterior
- Modeling an assembly chain

  use in CAD :

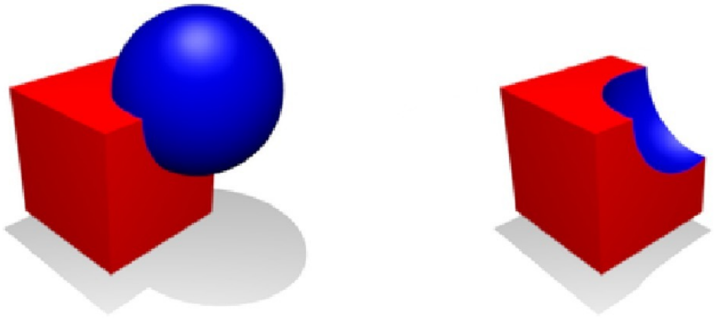
    Solid Works,
    AutoCAD,
    Catia,
    PovRay.



BRep VS CSG



BRep



CSG

+ Modeling complex object

- Approximation
- Surface only
- Discretization

Arbitrary discrete surfaces
(Graphics, Computation, CAD)

+ Exact
+ Constructive method

- Limited possibilities
- Teadious for complex shapes
- Non unique construction

Simple exact shapes
(CAO)

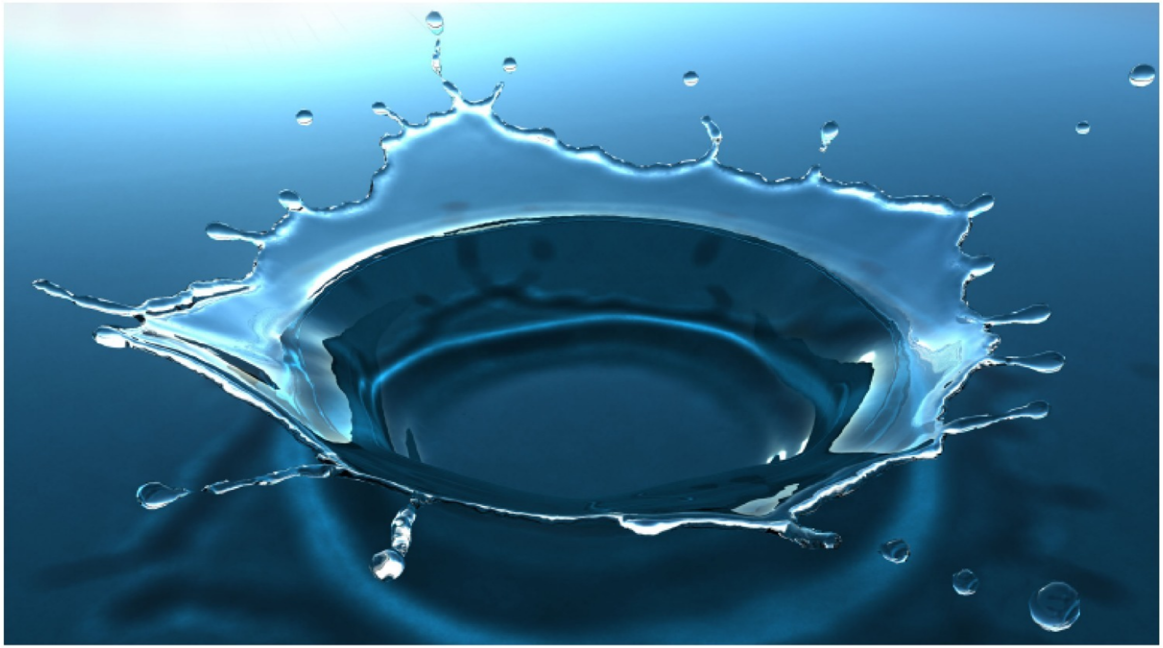
Implicit

Implicit Modeling

Why using implicit modeling: **Topology changes**

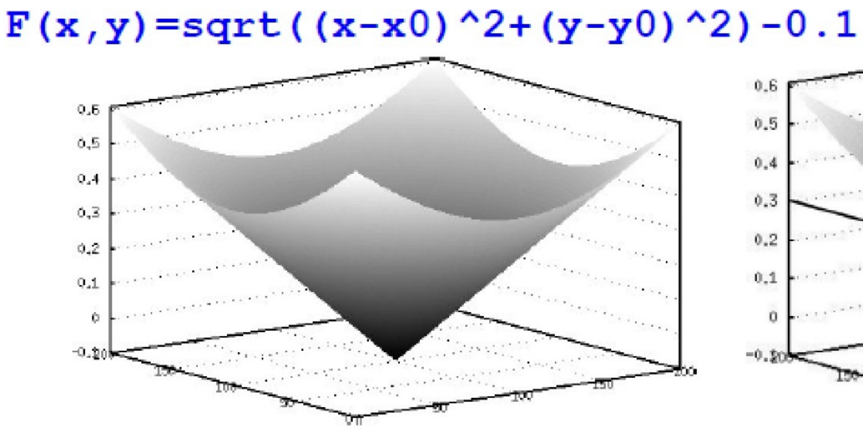
$$S = \{\mathbf{p} \in \mathbb{R}^3 \mid \psi(\mathbf{p}) = 0\}$$

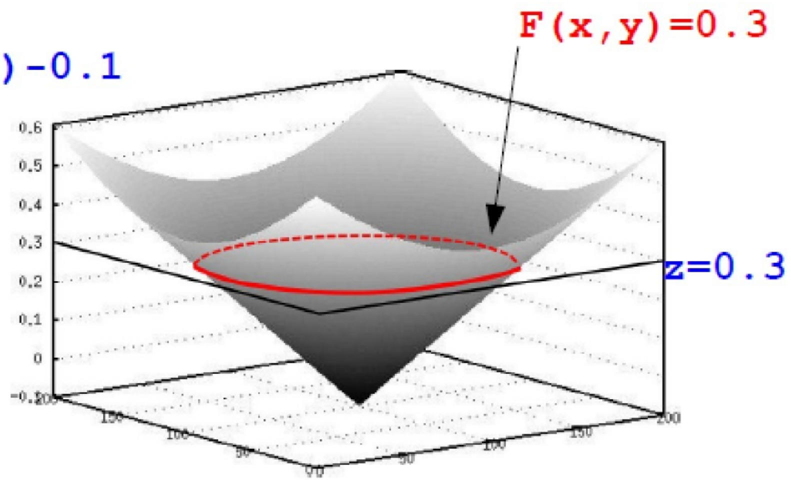
$$S = \psi^{-1}(0)$$

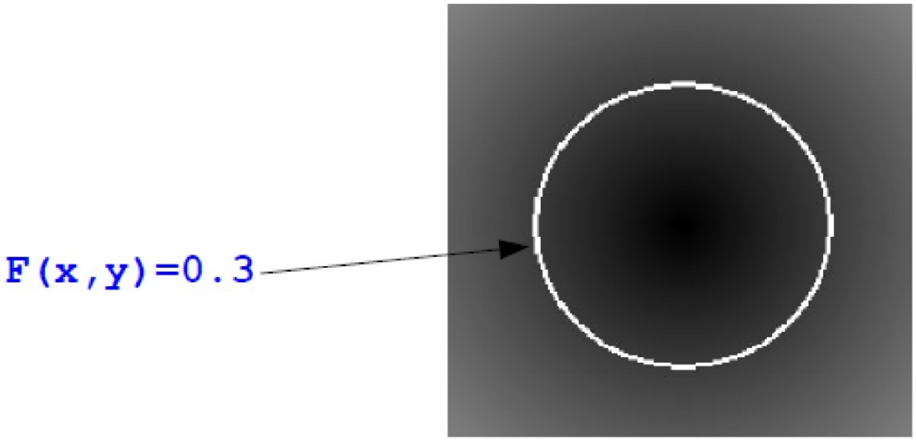


[Thuereu, Wojtan, Gross, Turk, SIGGRAPH 2010]

Implicit Modeling in 2D



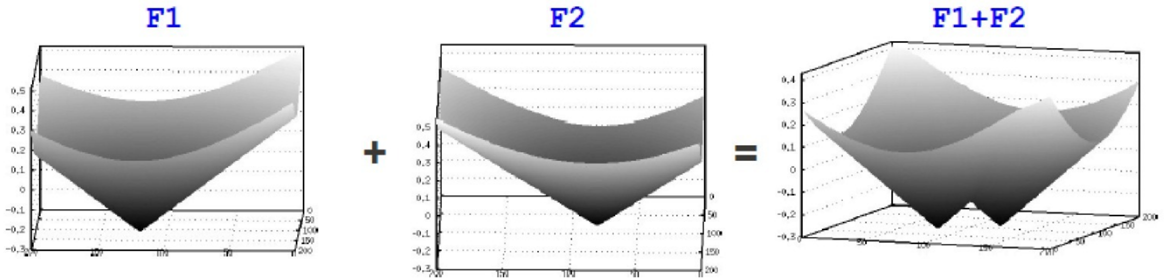


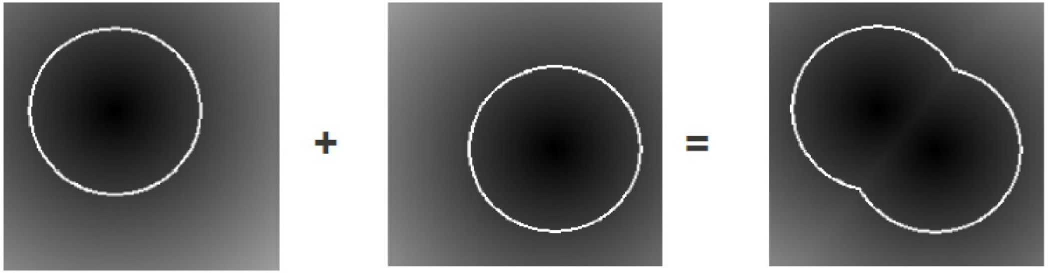


Implicit Modeling in 2D

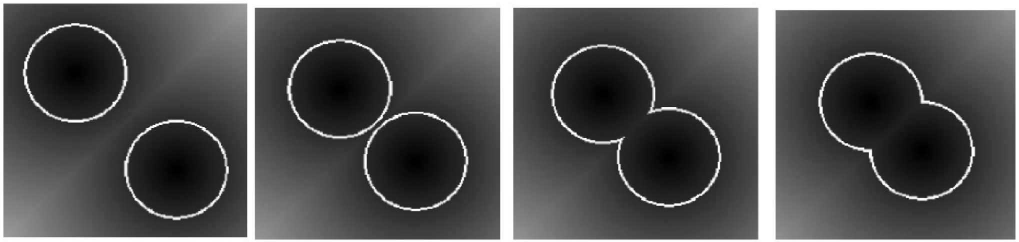
`F1(x,y)=sqrt((x-x1)^2+(y-y1)^2)-0.1`

`F2(x,y)=sqrt((x-x2)^2+(y-y2)^2)-0.1`



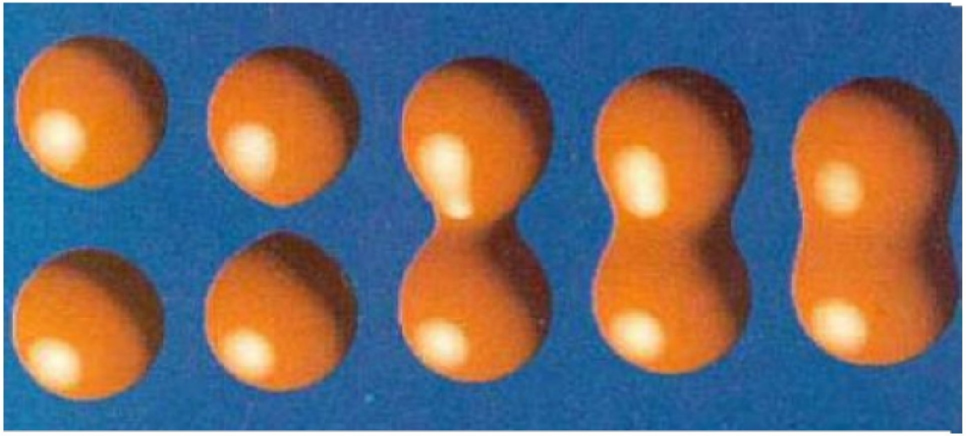


position of center: different topology

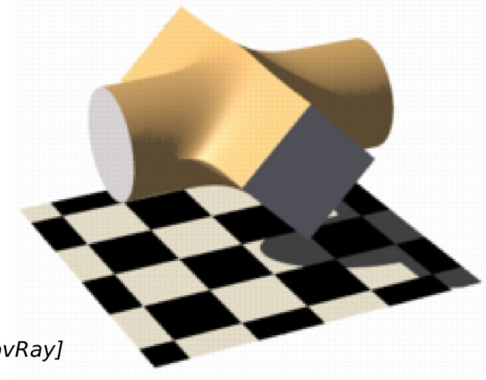


Implicit Modeling in 3D

$$\psi(x, y, z) = 0$$



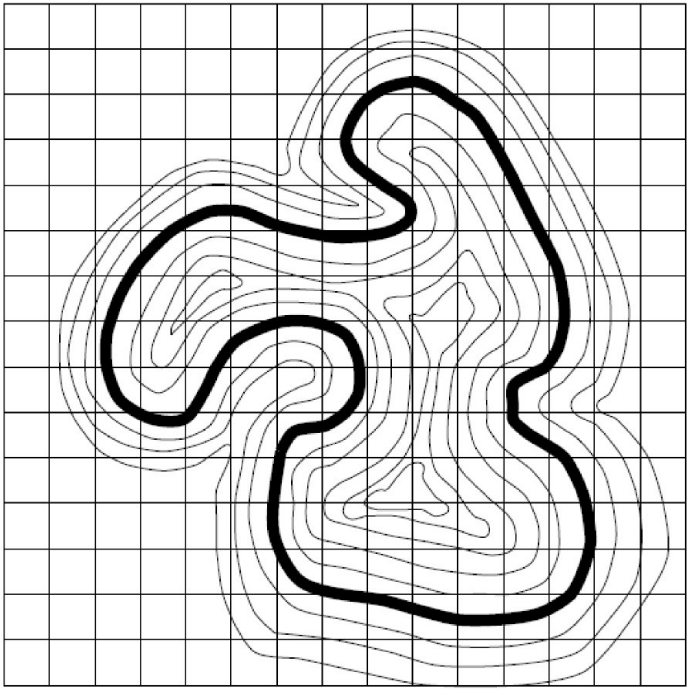
[Wyvill, McPheeters, Wyvill, The Visual Computer, 1986]



[PovRay]

Implicit Function Encoding

How do we encode the field function ?

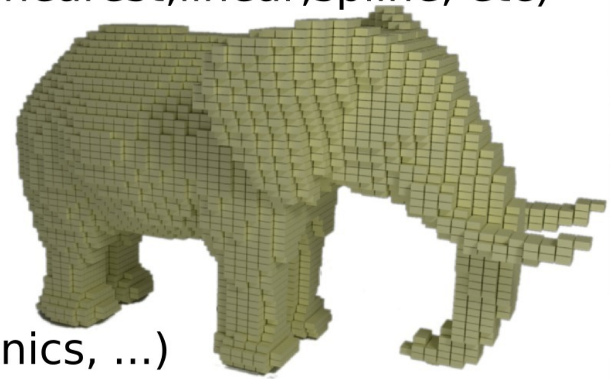


[Sethian]

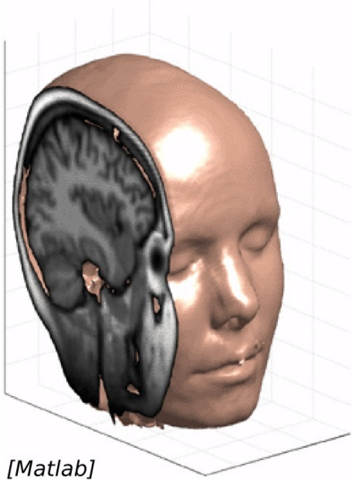
Voxel Grid

- Spatial discretization in voxels
- Store each value in the grid $\psi(k_x, k_y, k_z)$
- Access value using interpolation (nearest,linear,spline, etc)

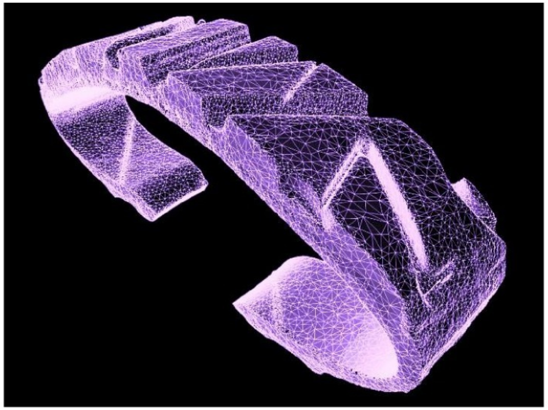
+ Very general
- Memory storage
  (1024^3 voxels => 8Go RAM)



Scanner imaging (medical, mecanics, ...)



[Matlab]

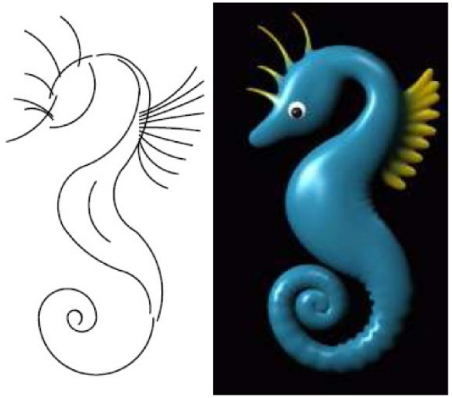


[Digisens]

Parametric

Skeleton based

- Blobs $\psi_i(\mathbf{p}) = e^{\frac{\|\mathbf{p}-\mathbf{p}_i\|^2}{\sigma^2}}$
- Metaballs $\psi_i(\mathbf{p}) = \sum_k \omega_k \|\mathbf{p}-\mathbf{p}_i\|^k$
- Convolution $\psi_i(\mathbf{p}) = \int_{\mathbf{q}\subset S_i} \omega(\mathbf{q})\, h(\|\mathbf{p}-\mathbf{q}\|)\, \mathrm{d}\mathbf{q}$

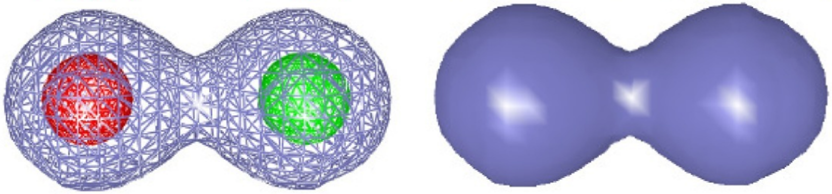


[Sherstyuk, 98-99]

Final field:
Sum of the contribution

$$\psi(\mathbf{p}) = \sum_i \psi_i(\mathbf{p})$$

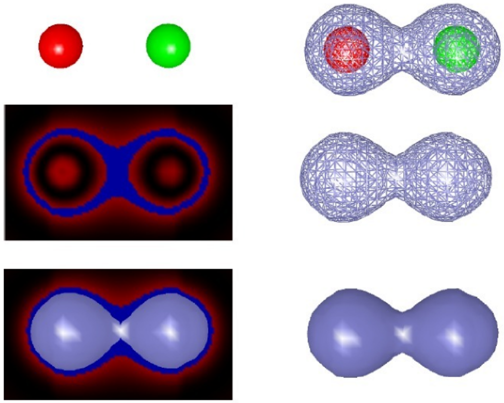
$\psi_1(\mathbf{p})$ $\psi_2(\mathbf{p})$ $\psi(\mathbf{p}) = \psi_1(\mathbf{p}) + \psi_2(\mathbf{p})$

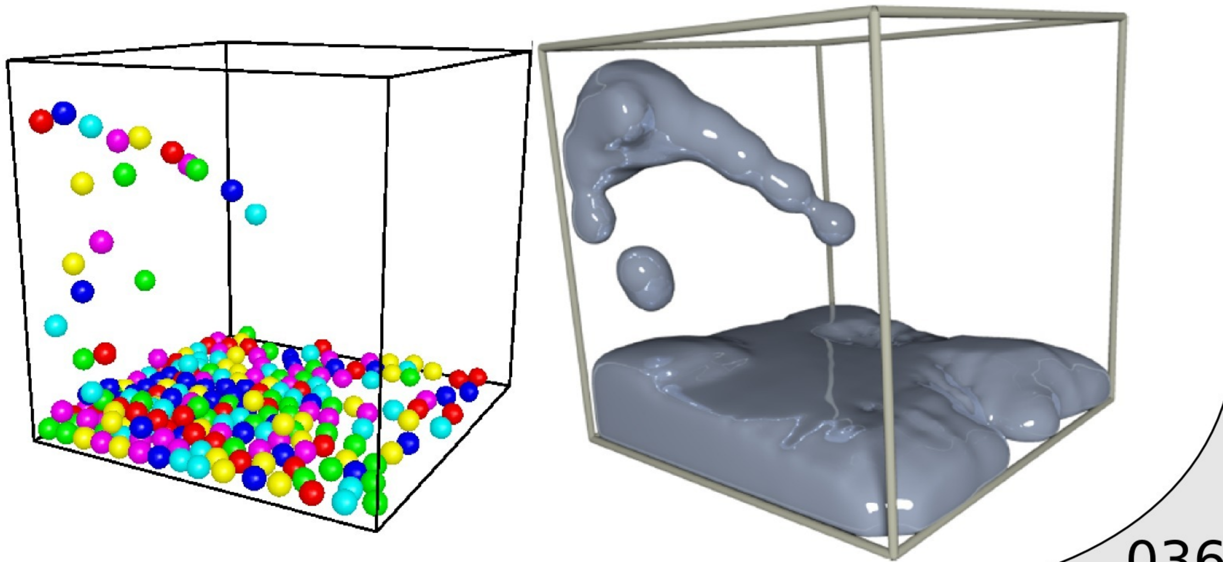


Direct manual control.
=> use for Graphics

Parametric

Skeleton based: Application to fluid simulation





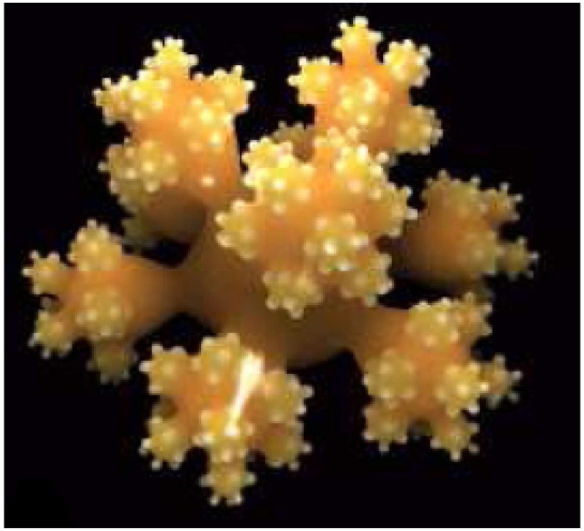
Parametric

Function opitimization/fitting

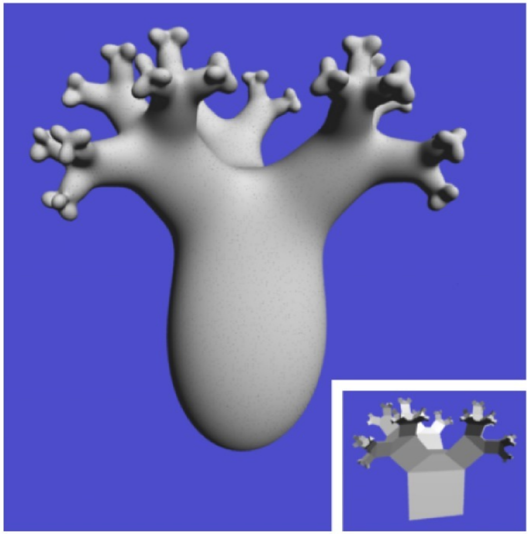
- Splines
- RBF

+ More general than skeleton based
- Requires a minimization, no direct control

=> mostely medical applications



[Sherstyuk, 98]

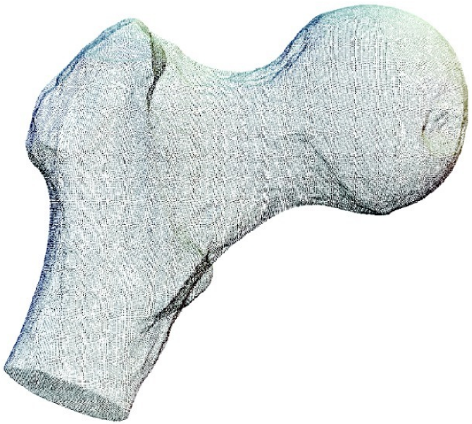


[Turk, O'Brien, SIGGRAPH 02]

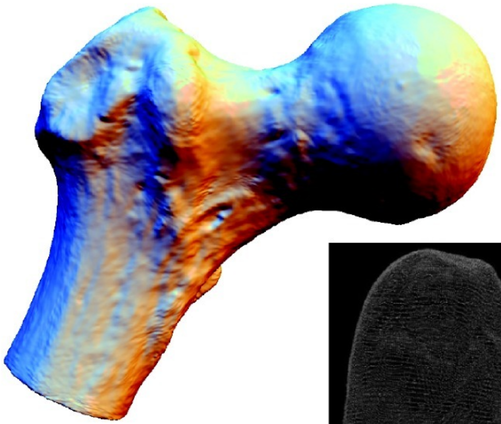
Points set

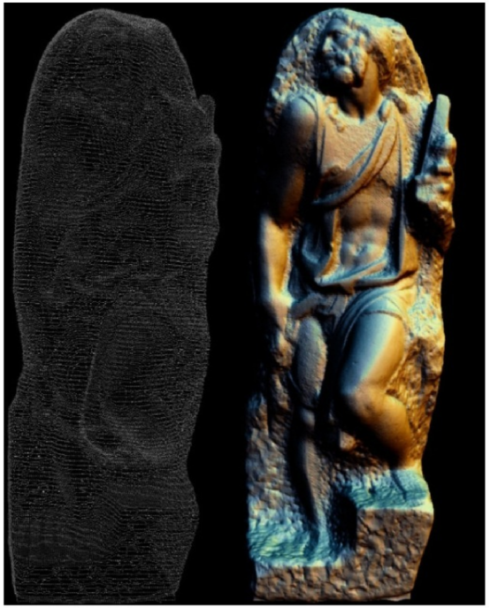
Input = discrete points (+ normals)

Data from scanners



[Boubekeur]





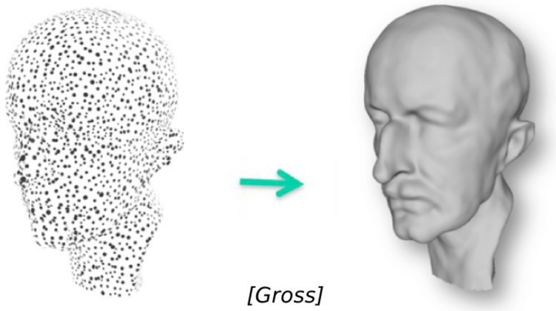
Points set

Moving Least Squares (MLS)

Goal: Find f, smooth function such that

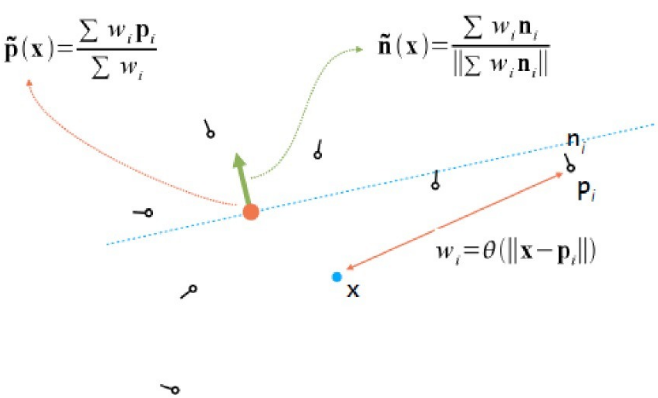
$$f = \operatorname{argmin} \left(\sum_i \psi(\|\mathbf{p} - \mathbf{p}_i\|) \; (f(\mathbf{p}) - f(\mathbf{p}_i))^2 \right)$$

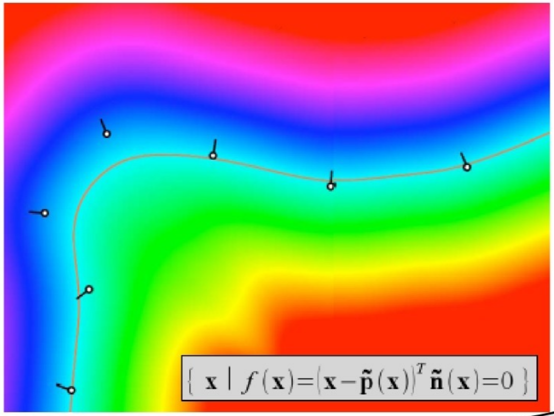
+ Smooth approximating functions
- Minimization



[Gross]

Application to noisy data





Points set

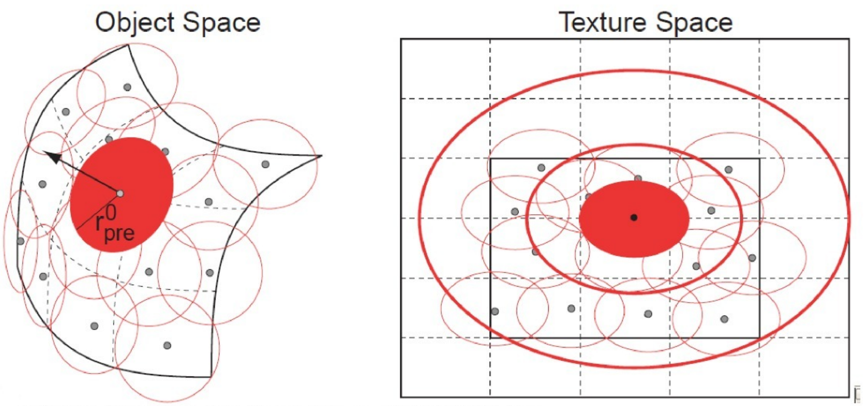
Surfels

Goal: Render a continuous surface from simple primitives

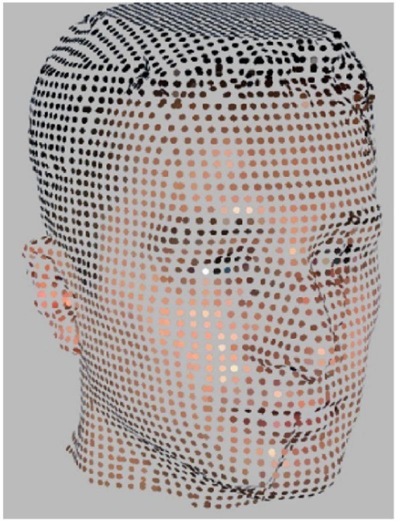
$$f = \operatorname{argmin}\left(\sum_i \psi(\|\mathbf{p}-\mathbf{p}_i\|)\ \left(f(\mathbf{p}) - f(\mathbf{p}_i)\right)^2\right)$$

+ Fast rendering
- No underlying surface

Application to big data



[Pfister, Zwicker, van Baar, Gross, SIGGRAPH 00]

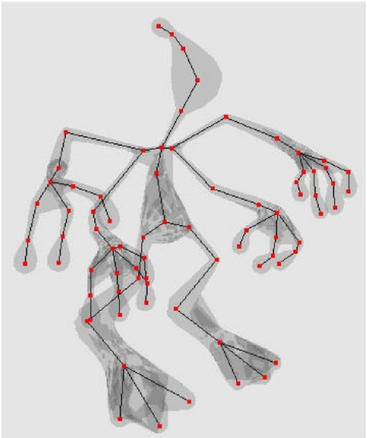


[Zwicker]

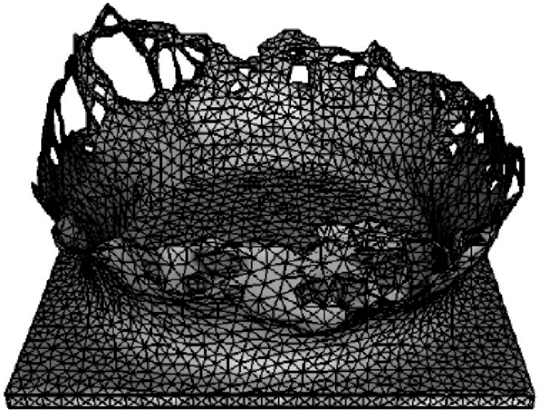
Implicit VS Explicit

+ Arbitrary topology
+ Shape blending

- Manipulation
- Memory cost
- Rendering time
- Surface details



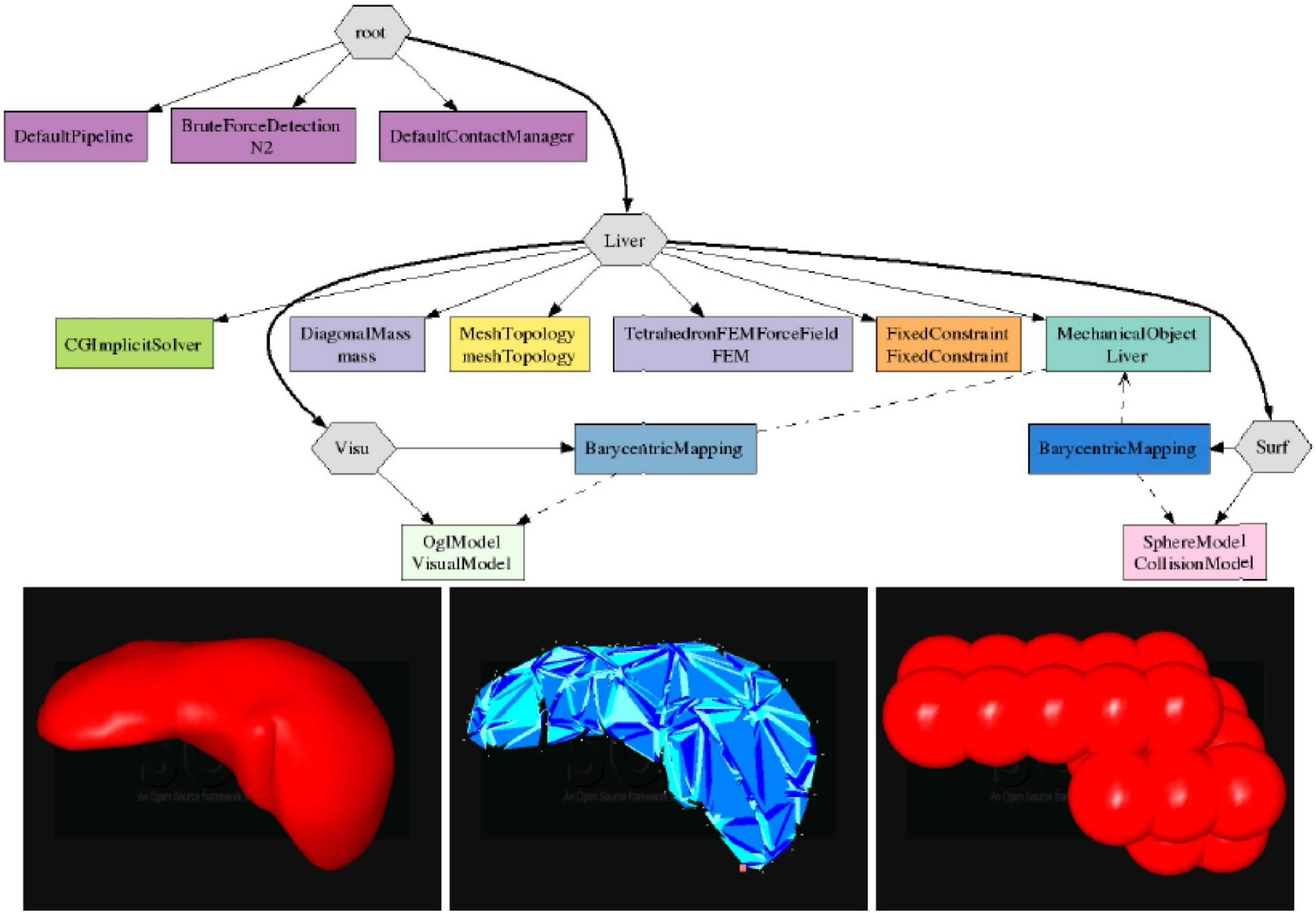
[Hornus, Angelidis, Cani, Vis. Comp. 03]



[Broshu, Batty, Bridson, SCA 09]

[Ohtake, Belyaev, Alexa, Turk, Seidel, SIGGRAPH 03]

Interaction between representations



[SOFA, Inria]

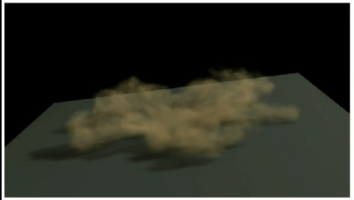
Fractals

Fractals

Principles:
Reccursive deformations converging toward a complex shape

Use:
Modeling complex objects from a few simple rules

Application: Graphics (procedural modeling)

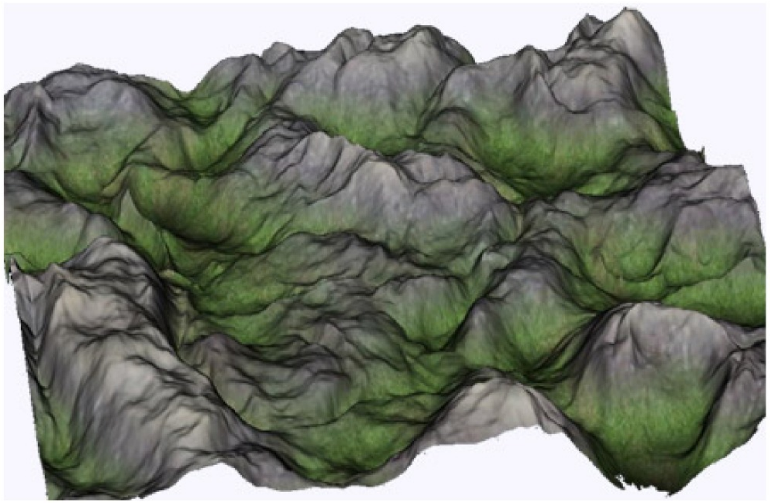
Perlin noise

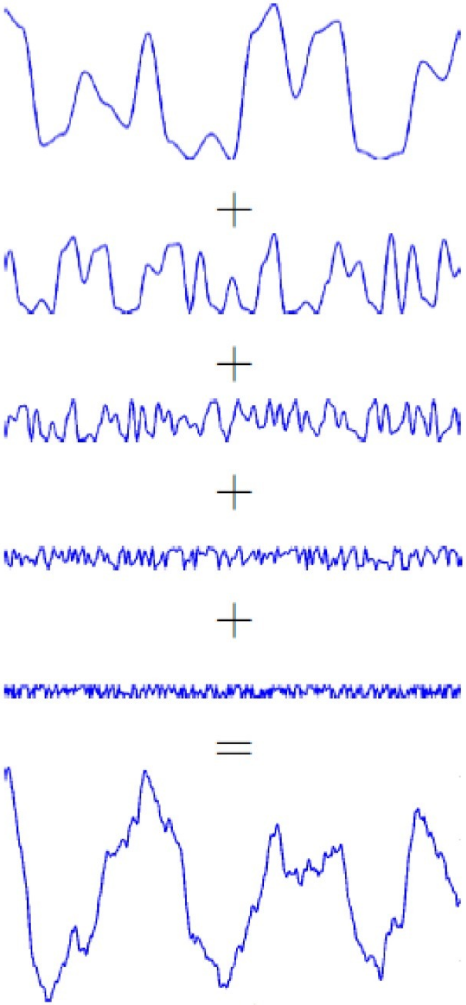
$$f(x) = \sum_{k=0}^{N} \alpha^k f(\omega^k x)$$

N : Octave

α : Attenuation (1/α: persitence)

ω : Frequence gain

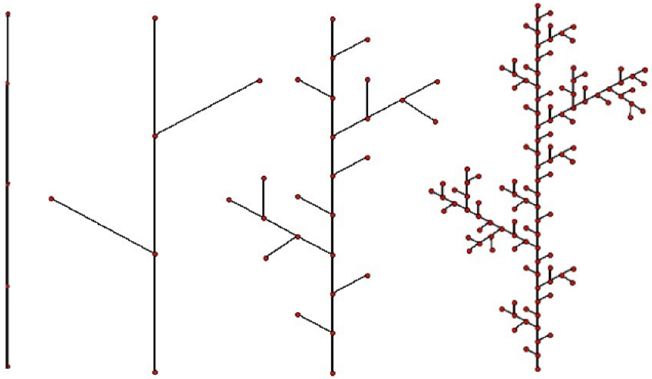




L-System

Grammar:

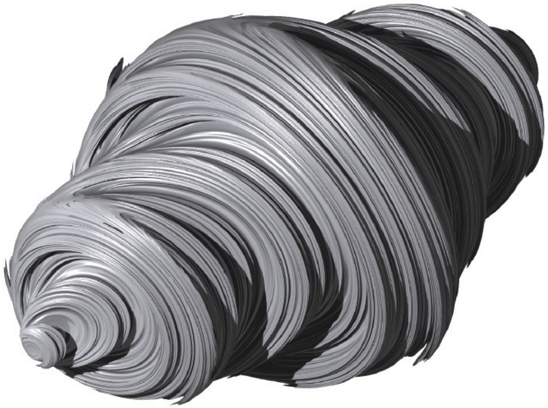
F [+F] F [-F] F , $\theta = 60^\circ$

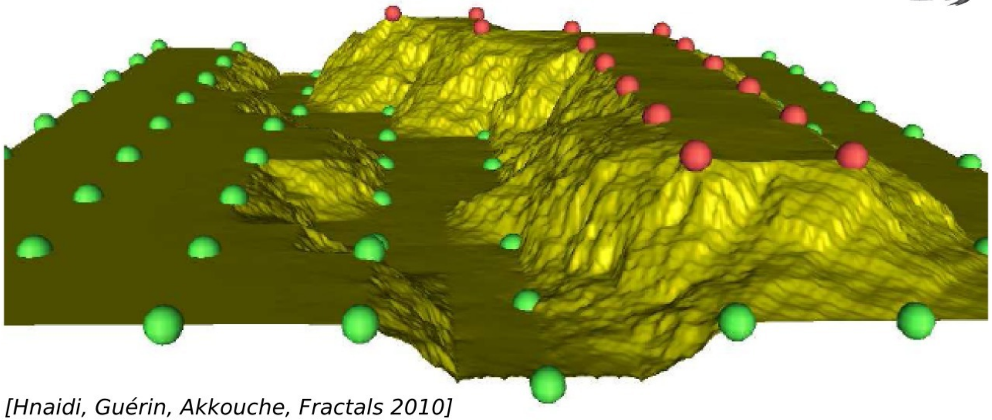


Fractals, conclusion

+ Complex objects from simple rules
+ Natural aspect

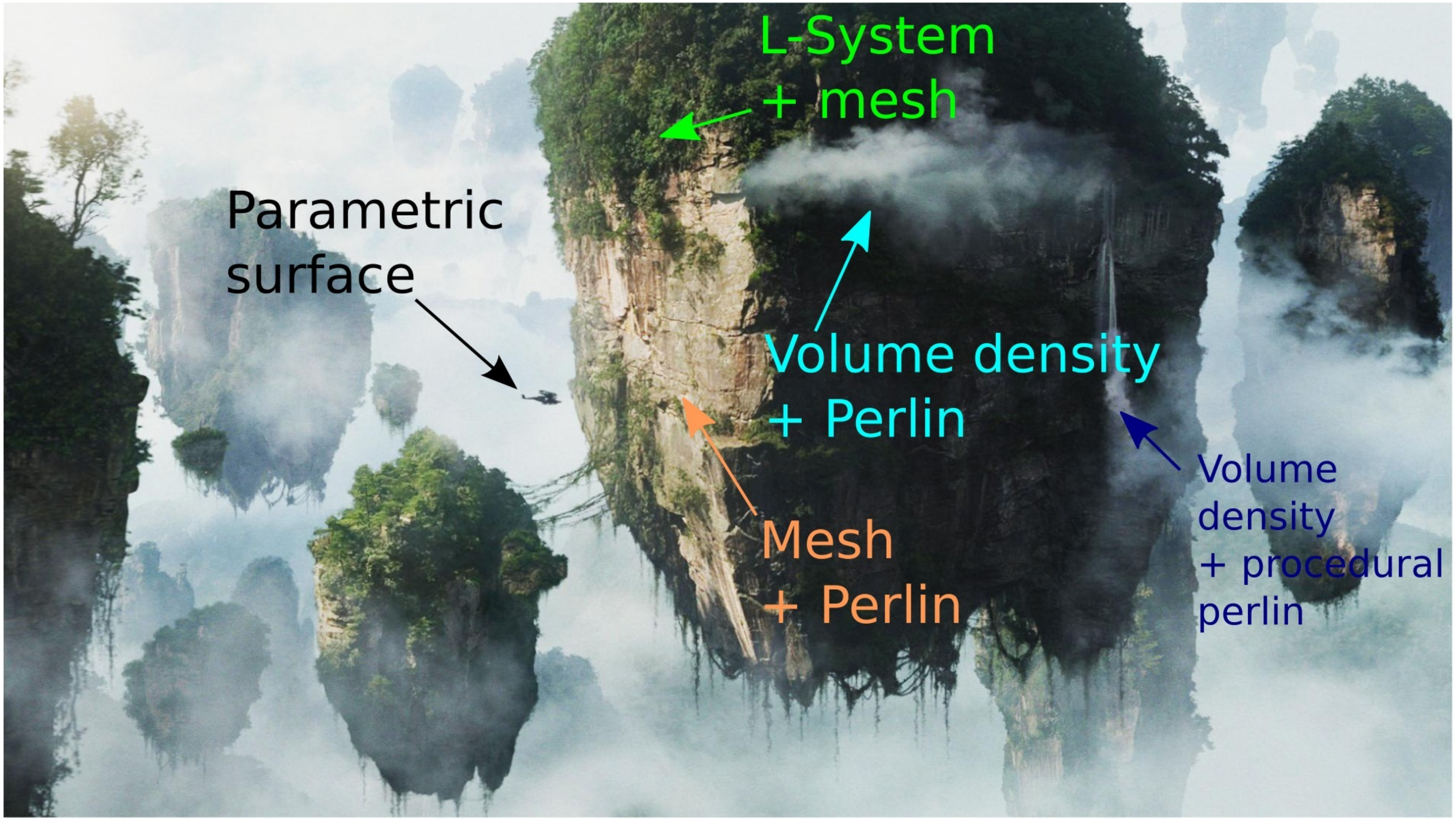
- Control





[Hnaidi, Guérin, Akkouche, Fractals 2010]

Various modeling



[Avatar]