5ETI Synthèse d'images: Maillage

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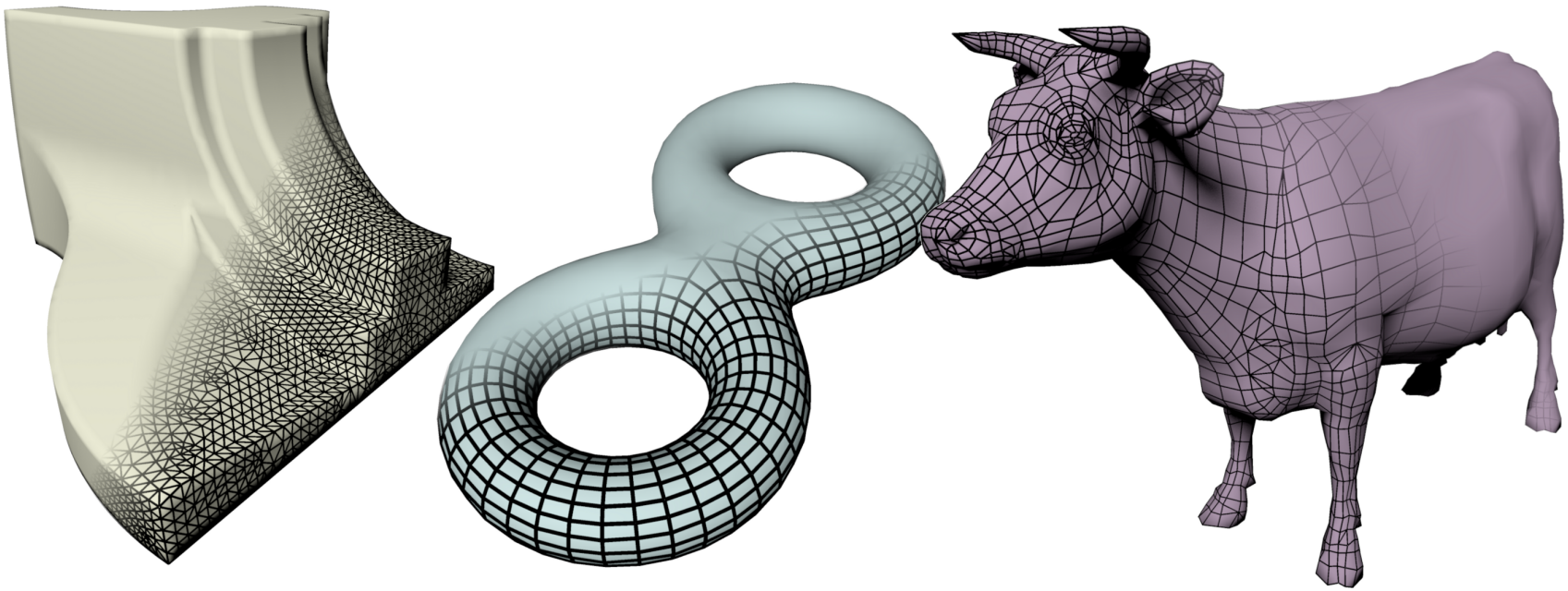
Plan du cours

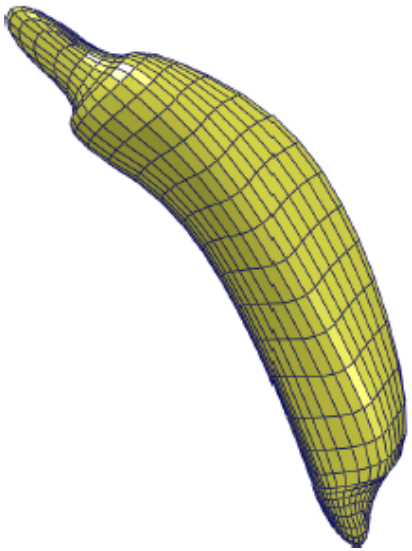
Maillage

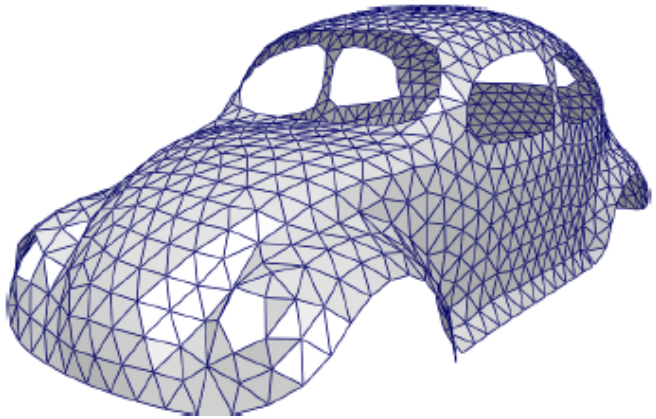
- Maillage (Mesh) = Ensemble de **polygones** partageants certains sommets
- N_f **faces**, N_s **sommets** (vertices), N_e **arêtes** (edges).
- **Triangulation** : toutes les faces sont des triangles.
- **Quad-mesh** : toutes les faces sont des quads.
- Poly-mesh : mélange de types.

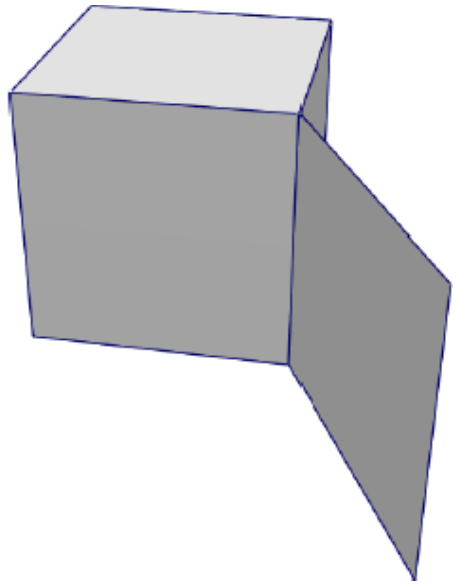


Topologie

- Rappel : Surface variété (Manifold) ssi le voisinage de tout point est homéomorphe à un (demi) disque.
  $\Rightarrow$ Toute arête est partagée par au plus 2 faces (connectivitée) + non auto-intersection (plongement).







Maillage

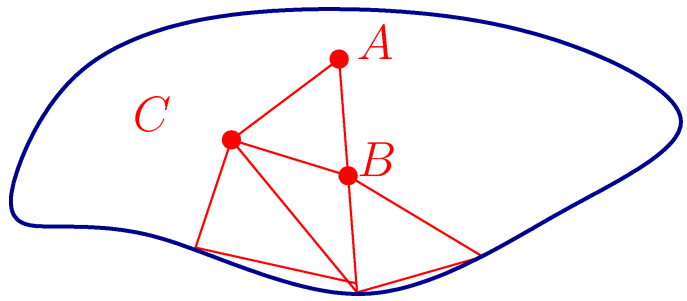
- Poly-mesh = cas particulier de triangulation
- Rappel : Triangulation = Mapping linéaire S

$$S_i : \begin{cases} \mathcal{D} \subset \mathbb{R}^2 & \to \mathbb{R}^3 \\ (u, v) & \mapsto S_i(u, v) = u\overrightarrow{AB} + v\overrightarrow{AC} \end{cases}$$

$$\mathcal{D} : 0 \leq u + v \leq 1$$

Propriétés :

- Surface globalement $\mathcal{G}^0$.
- Surface jamais $\mathcal{G}^1$ (sauf plan).
- **Interpolation linéaire** de $\mathbb{R}^2$ vers $\mathbb{R}^3$ (normales, couleurs, textures).



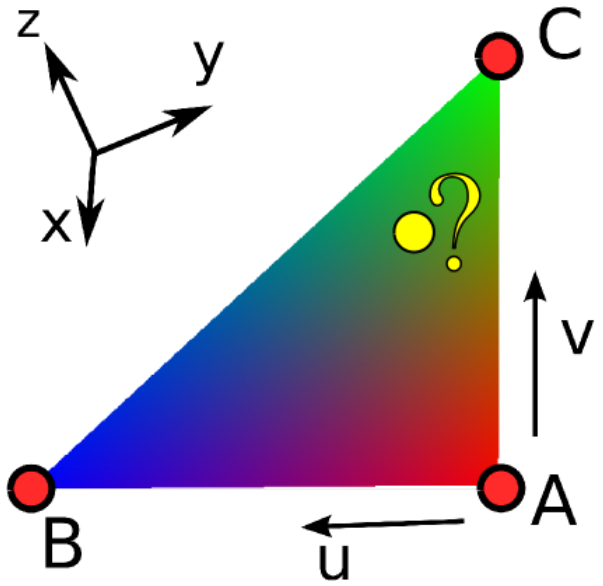
Coordonnées dans un triangle

- Position du point **p** par rapport aux sommets (A,B,C) ?

$$\overrightarrow{AP} = u\,\overrightarrow{AB} + v\,\overrightarrow{AC}$$

$$\Rightarrow P - A = u\,(B - A) + v\,(C - A)$$

$$\Rightarrow P = \underbrace{(1 - u - v)}_{w}\,A + u\,B + v\,C\ .$$



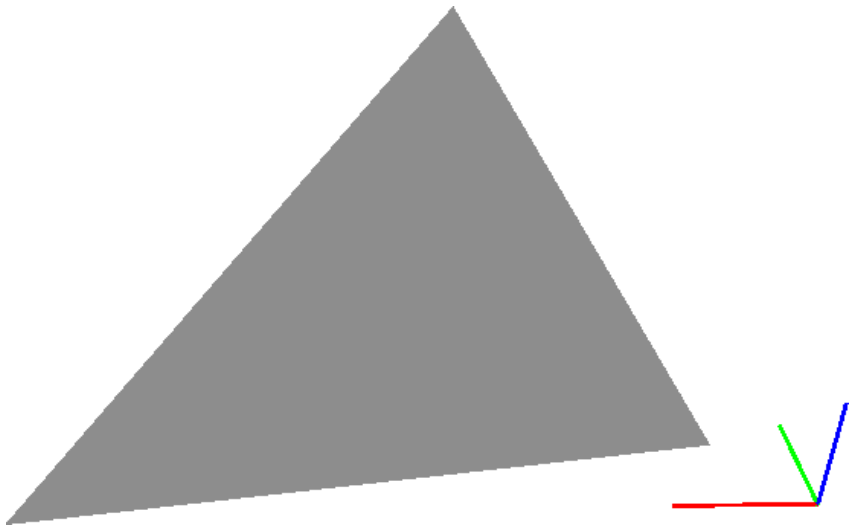
  - (u,v,w)=Coordonnées barycentriques

$$\begin{cases} P = w\,A + u\,B + v\,C \\ u + v + w = 1 \\ 0 \leq (u, v, w) \leq 1\ . \end{cases}$$

Triangles en OpenGL

- Syntaxe OpenGL (mode immédiat)

```
glBegin(GL_TRIANGLES);
glNormal3d(0,0,1);
glVertex3d(0,0,0);
glVertex3d(1,0,0);
glVertex3d(0,1,0);
glEnd();
```



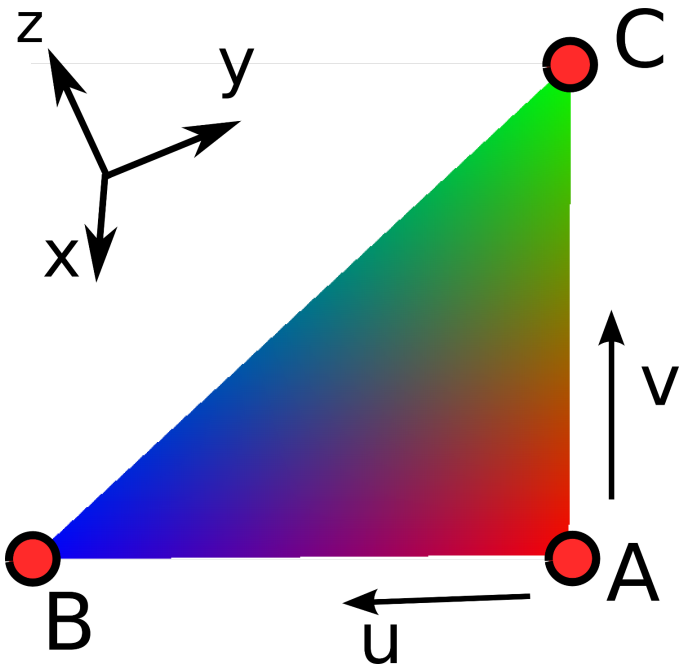
Interpolation linéaire

- Interpolation de couleurs

$$\begin{cases} r(u,v) & = (1-u-v)r_A + ur_B + vr_C \\ g(u,v) & = (1-u-v)g_A + ug_B + vg_C \\ b(u,v) & = (1-u-v)b_A + ub_B + vb_C \end{cases}$$

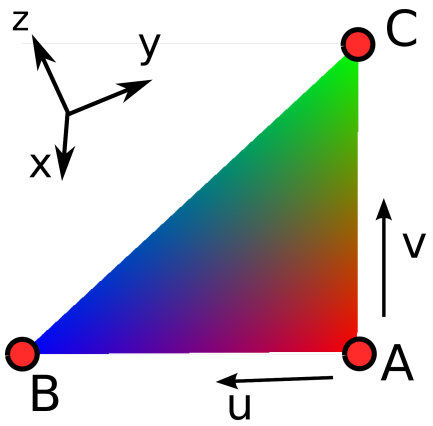
- Dans le cas général pour une fonction f

$$f(u,v) = (1-u-v)f_A + uf_B + vf_C$$



Interpolation de couleurs

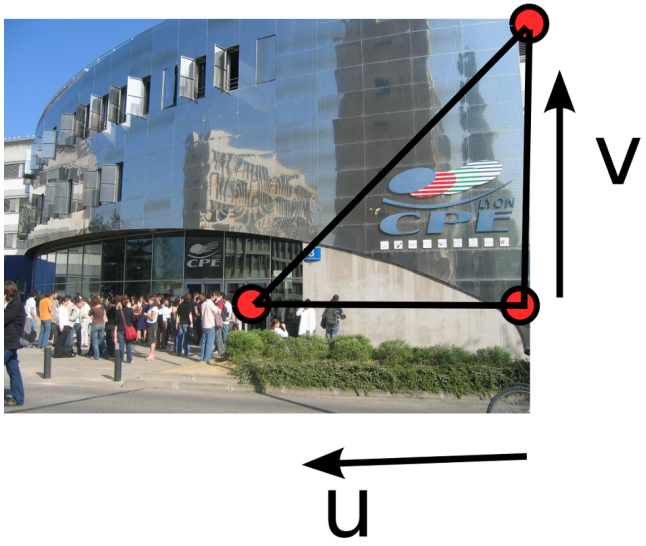
```
glBegin(GL_TRIANGLES);
glNormal3d(0,0,1);
glColor3d(1,0,0);
glVertex3d(0,0,0);
glColor3d(0,1,0);
glVertex3d(1,0,0);
glColor3d(0,0,1);
glVertex3d(0,1,0);
glEnd();
```

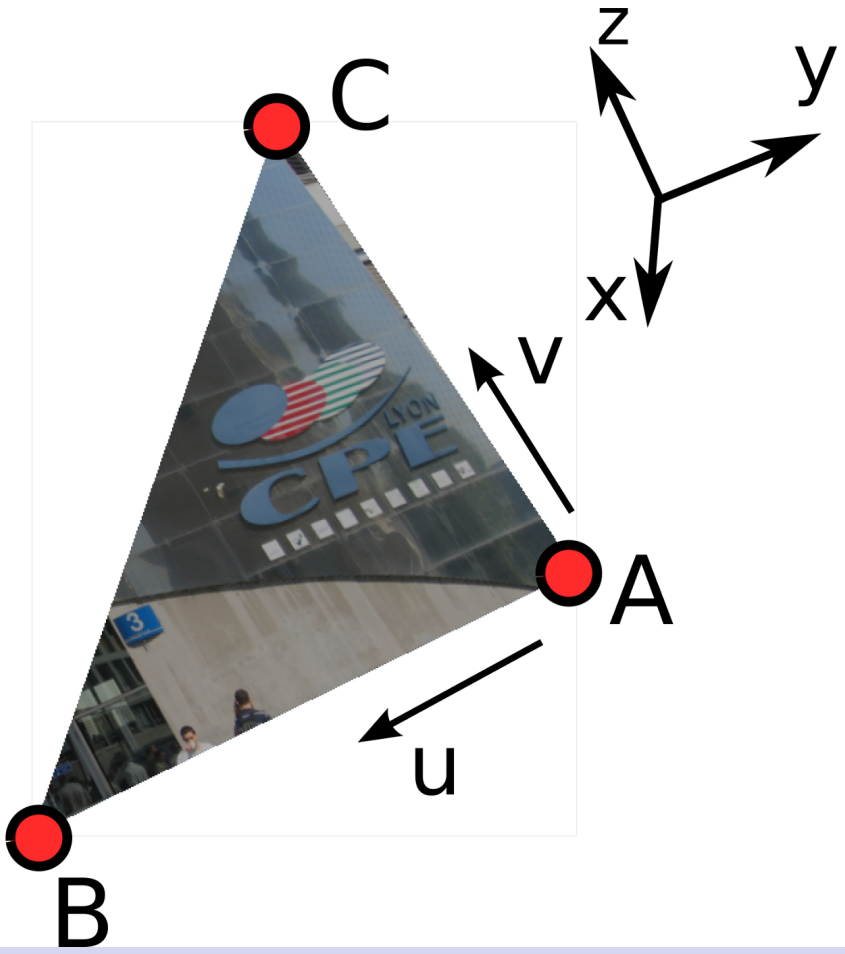


Interpolation linéaire

- On peut interpoler des coordonnées !
  $\Rightarrow$ Textures

$$\begin{cases} t_x = (1-u-v)\,t_x(A) + u\,t_x(B) + v\,t_x(C) \\ t_y = (1-u-v)\,t_y(A) + u\,t_y(B) + v\,t_y(C) \end{cases}$$





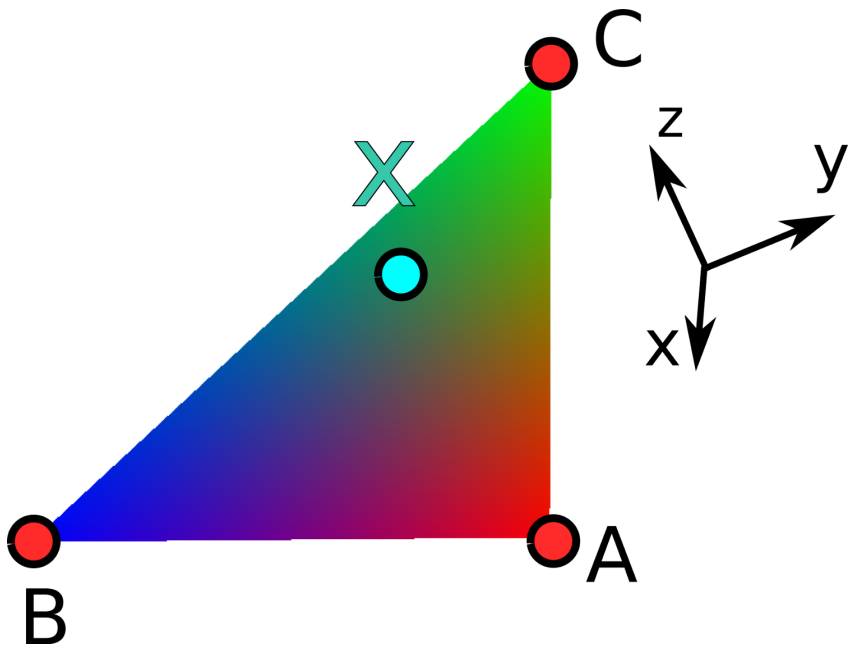
Coordonnées barycentriques

- Étant donné un point $\mathbf{x} = (x, y, z) \in \mathbb{R}^3$: connaitre (α, β, γ) tel que $\mathbf{x} = \alpha\mathbf{x}_A + \beta\mathbf{x}_B + \gamma\mathbf{x}_C$, $(\alpha + \beta + \gamma = 1)$.
  $\Rightarrow$ coordonnées barycentriques.

$$\begin{cases} A = aire(\mathbf{x}_B - \mathbf{x}_A, \mathbf{x}_C - \mathbf{x}_A) \\ A_1 = aire(\mathbf{x}_C - \mathbf{x}_B, \mathbf{x} - \mathbf{x}_B) \\ A_2 = aire(\mathbf{x}_A - \mathbf{x}_C, \mathbf{x} - \mathbf{x}_C) \\ A_3 = aire(\mathbf{x}_B - \mathbf{x}_A, \mathbf{x} - \mathbf{x}_A) \end{cases}$$

avec $aire(\mathbf{v}_0, \mathbf{v}_1) = 1/2\|\mathbf{v}_0 \times \mathbf{v}_1\|$

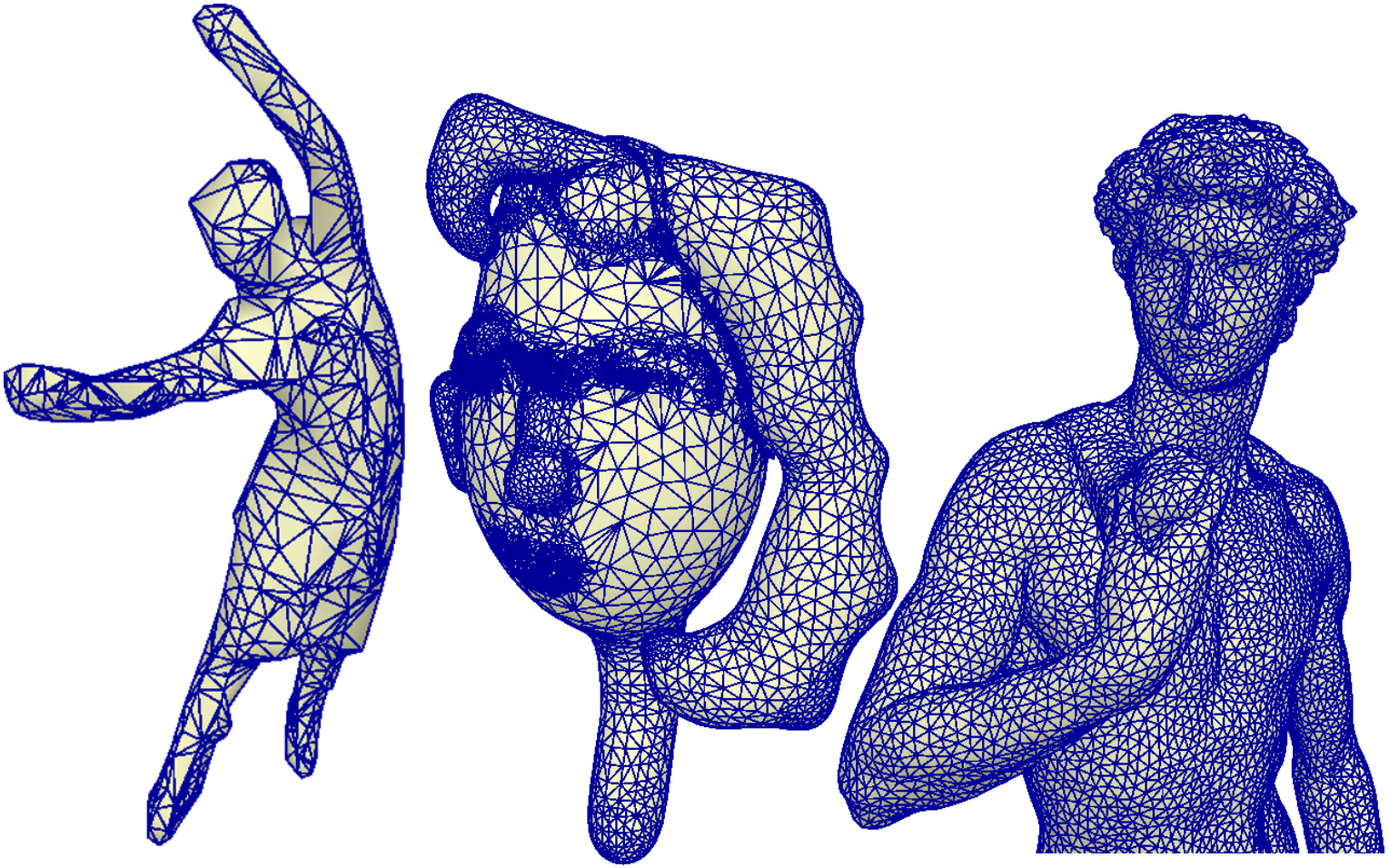
$$\Rightarrow \begin{cases} \alpha = A_1/A \\ \beta = A_2/A \\ \gamma = A_3/A \end{cases}$$



Qualité d'un maillage

- Triangulation : $\theta_{\text{min}} \simeq 30°$
- Quads : $\theta_{\text{min}} \simeq 45°$

Application : Calculs (FEM), (Rendu)



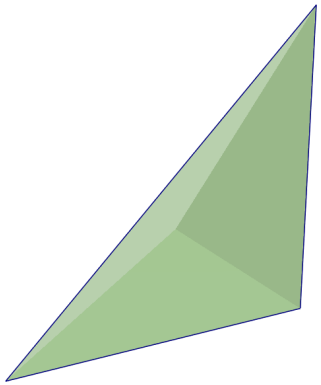
Maillage

Structure de données :

- *Ex. Représenter un tetraèdre* :

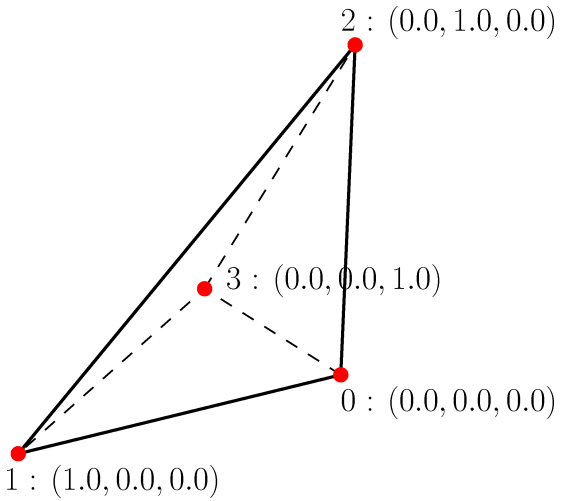
Idée 1 :

```
(0.0,0.0,0.0),(1.0,0.0,0.0),(0.0,0.0,1.0)
(0.0,0.0,0.0),(0.0,0.0,1.0),(0.0,1.0,0.0)
(0.0,0.0,0.0),(0.0,1.0,0.0),(1.0,0.0,0.0)
(0.0,1.0,0.0),(0.0,0.0,1.0),(1.0,0.0,0.0)
```



Il y a mieux :

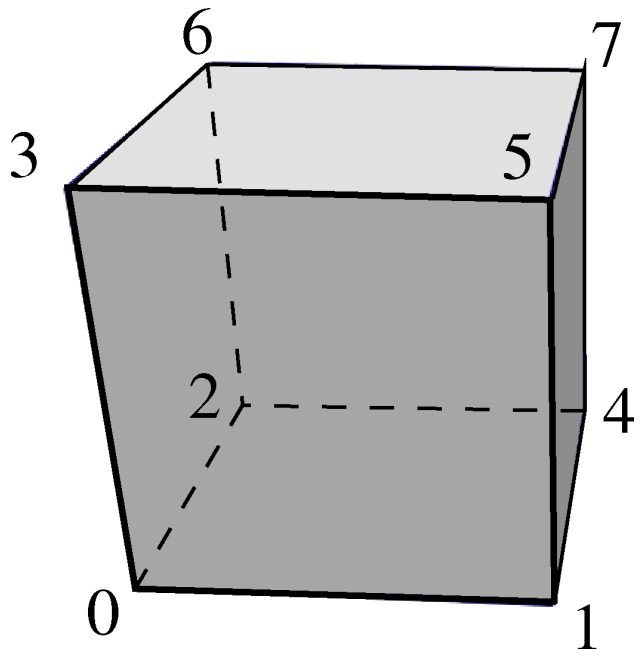
```
coordonnees:
(0,0,0),(1,0,0),(0,1,0),(0,0,1)
Connectivite
(0,1,3)
(0,3,2)
(0,2,1)
(1,2,3)
```



Format off

Exemple de format d'echange. Format off.

```
OFF
8  6  12
0  0  0
1  0  0
0  1  0
0  0  1
1  1  0
1  0  1
0  1  1
1  1  1
4  0  1  4  2
4  1  5  7  4
4  3  6  7  5
4  2  6  3  0
4  2  4  7  6
4  0  3  5  1
```



Lecture fichier

```
while(k_vertex<N_vertex)
  {
    fscanf(fid,"%f %f %f",X,X+1,X+2);
    add_vertex(X[0],X[1],X[2]);
    k_vertex++;
  }
for(k_poly=0;k_poly<N_poly;k_poly ++)
  {
    fscanf(fid,"%d",&size_poly);
    std::vector v_poly;
    for(k=0;k<size_poly;k++)
      {
        fscanf(fid,"%d",&temp);
        v_poly.push_back(temp);
      }
    add_polygon(v_poly);
  }
```

Structure de données

- Vecteurs contigüs dans la mémoire : Affichage rapide en openGL.

```
//(x0,y0,z0,x1,y1,z1,...)
std::vector <double> vertex

//(i00,i01,i02,i10,i11,i12,...)
std::vector <int> connectivity

std::vector <double> normal, color, texture ...
```

- Accès à la coordonnée *y* du sommet *k*.

  `vertex[3*k+1]`

- Accès à la coordonnée *y* du sommet s(1,2 ou 3) du triangle *t*.

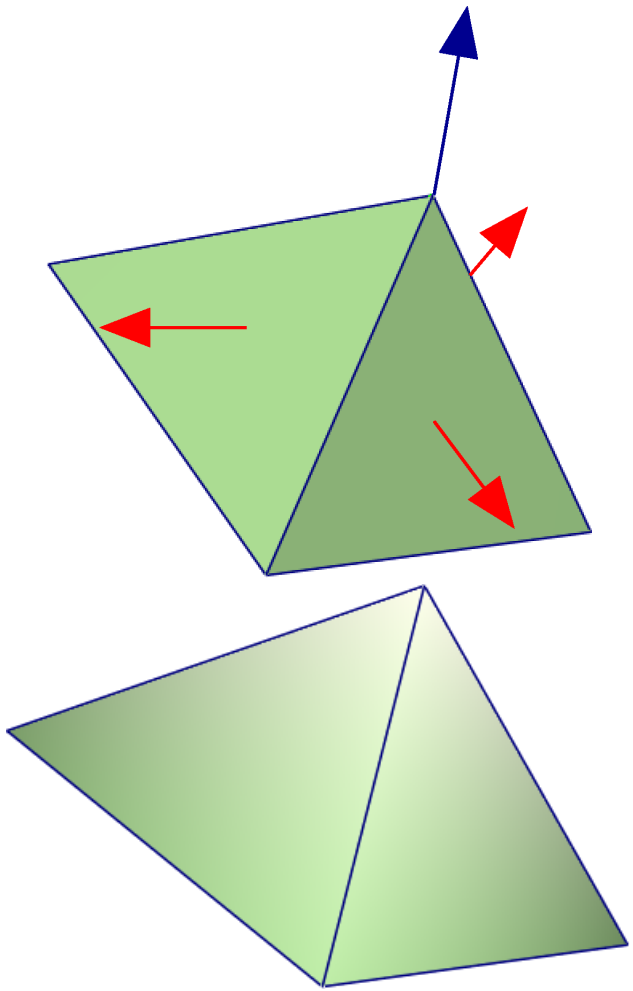
  `vertex[3*connectivity[3*t+s]+1]`

Normale d'un maillage

- Aspect lisse
  $\Rightarrow$ 1 normale par sommet.
- Moyenne de normales (faux mais répandue)

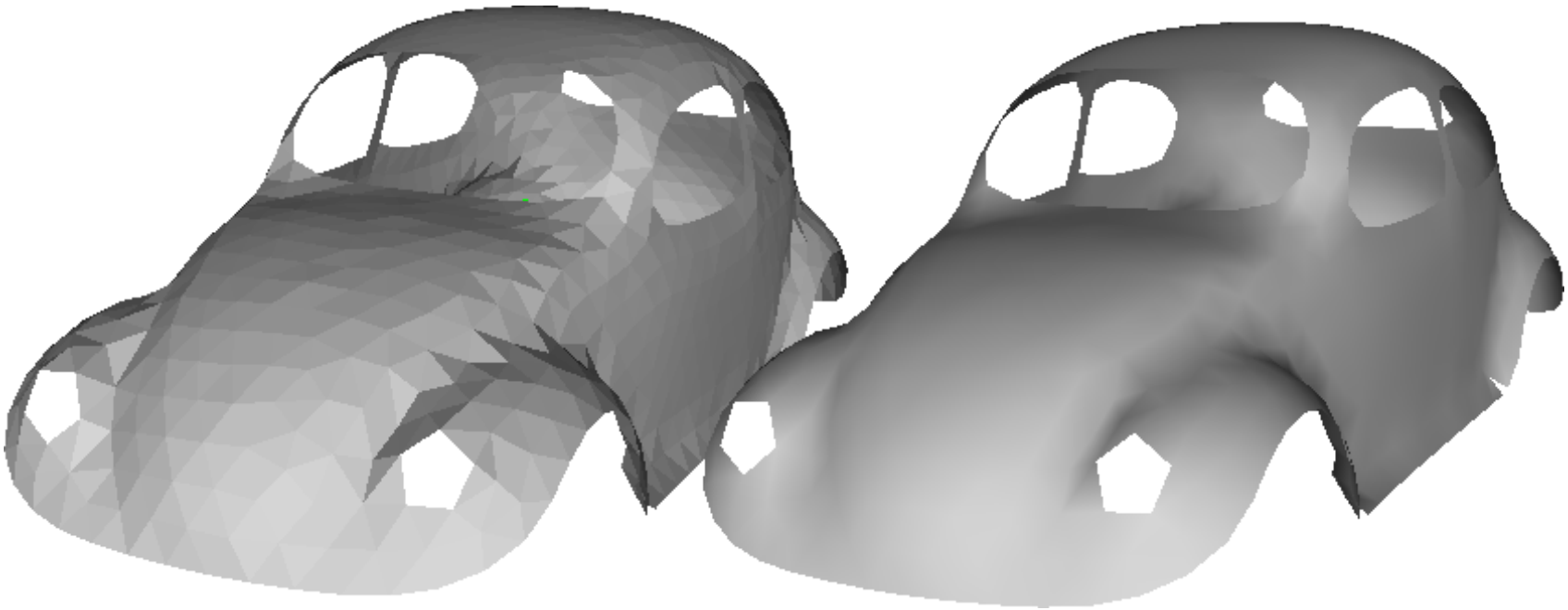
$$\mathbf{n}_k = \frac{\sum_{i \in \mathcal{V}(k)} \mathbf{n}_i}{\left\| \sum_{i \in \mathcal{V}(k)} \mathbf{n}_i \right\|}$$

k : indice sommet
i : indice face
$\mathcal{V}(k)$: faces voisines du sommet k



Normales

- En OpenGL : 1 Normale interpolé par sommets
  $\Rightarrow$ Normale par polygone = Plusieurs normales par sommet



Affichage en OpenGL

Version lente

```
glBegin(GL_TRIANGLES);
 for(k_tri=0;k_tri<N_tri;k_tri++)
  for(k_vertex=0;k_vertex<3;k_vertex++)
   for(k_dim=0;k_dim<3;k_dim++)
   {
    x[k_dim] = vertex[3*connectivity
                      [3*k_tri+k_vertex]+k_dim];
    n[k_dim] = normal[3*connectivity
                      [3*k_tri+k_vertex]+k_dim];

    glNormal3d(n[0],n[1],n[2]);
    glVertex3d(x[0],x[1],x[2]);
   }
glEnd();
```

Affichage en OpenGL

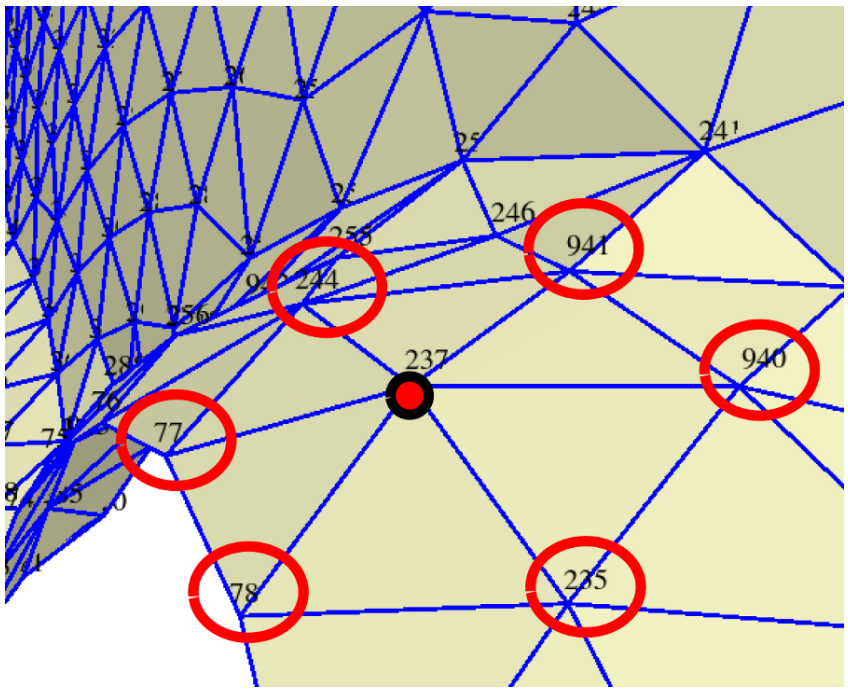
Version Rapide

```
glEnableClientState(GL_VERTEX_ARRAY);
glVertexPointer(3,GL_DOUBLE,0,&vertex[0]);

glEnableClientState(GL_NORMAL_ARRAY);
glNormalPointer(GL_DOUBLE,0,&normal[0]);

glDrawElements(GL_TRIANGLES,3*N_tri,
                    GL_UNSIGNED_INT,&connectivity[0]);

glDisableClientState(GL_VERTEX_ARRAY);
glDisableClientState(GL_NORMAL_ARRAY);
```

Structure de données : Voisinage

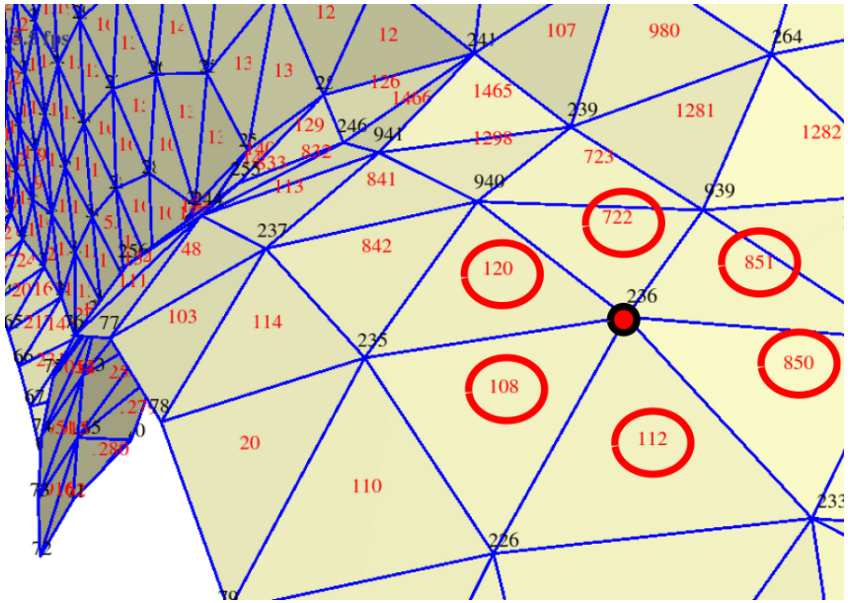
- **1-Voisinage** = Sommets voisins d'un sommet donné

```
std::vector <std::vector <int> > one_ring

// exemple pour le cube:
one_ring[0] = [1,2,3]
one_ring[1] = [5,4,0]
...
```



Voisinage

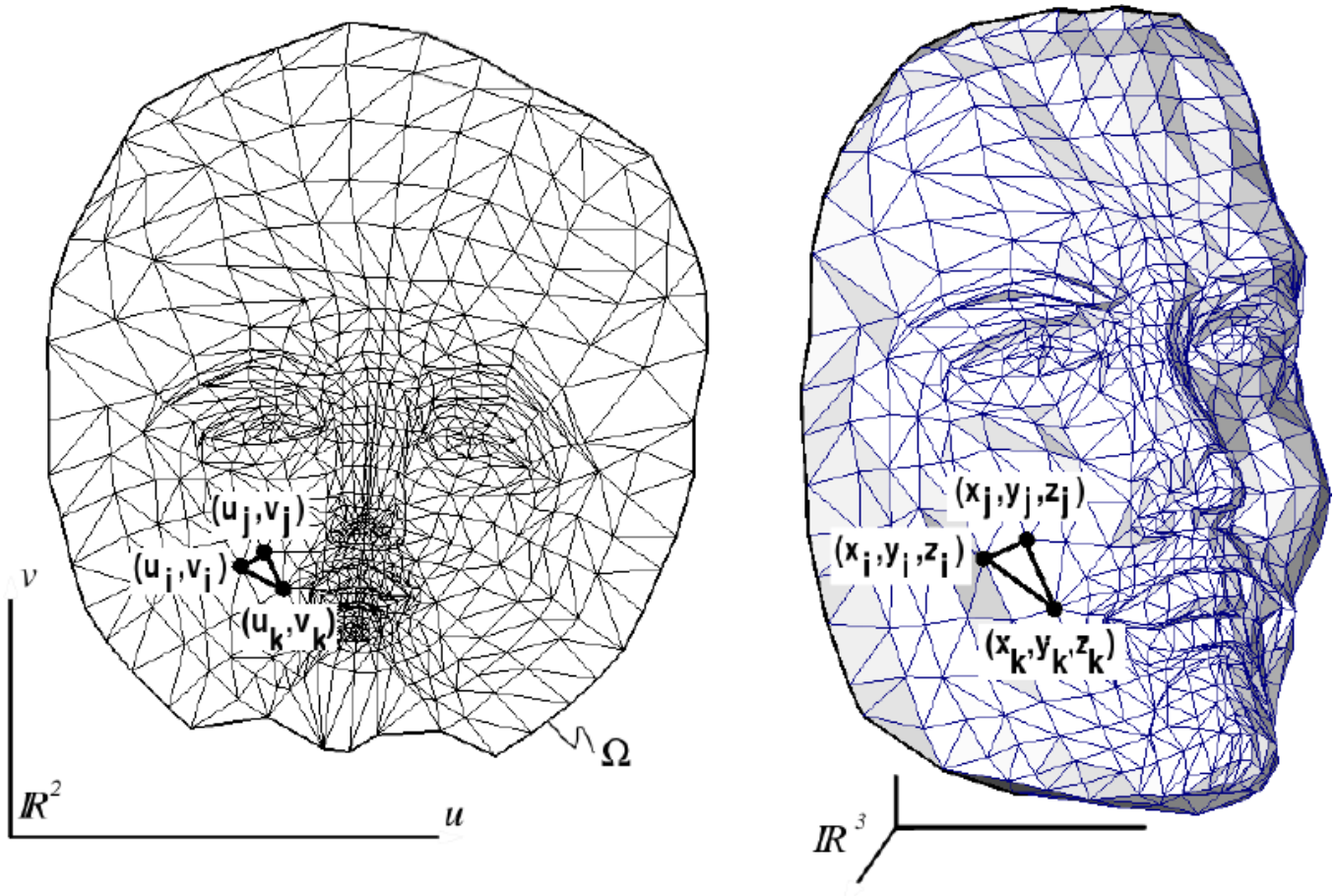
- Triangles voisins d'un autre
- Triangles voisins d'un point : etoilé (1-star)
  $\Rightarrow$ calcul des normales !



! Attention aux structures de données.
Compromis entre : temps accès / temps recherche / espace memoire / facilité ...

Parametrisation / Textures

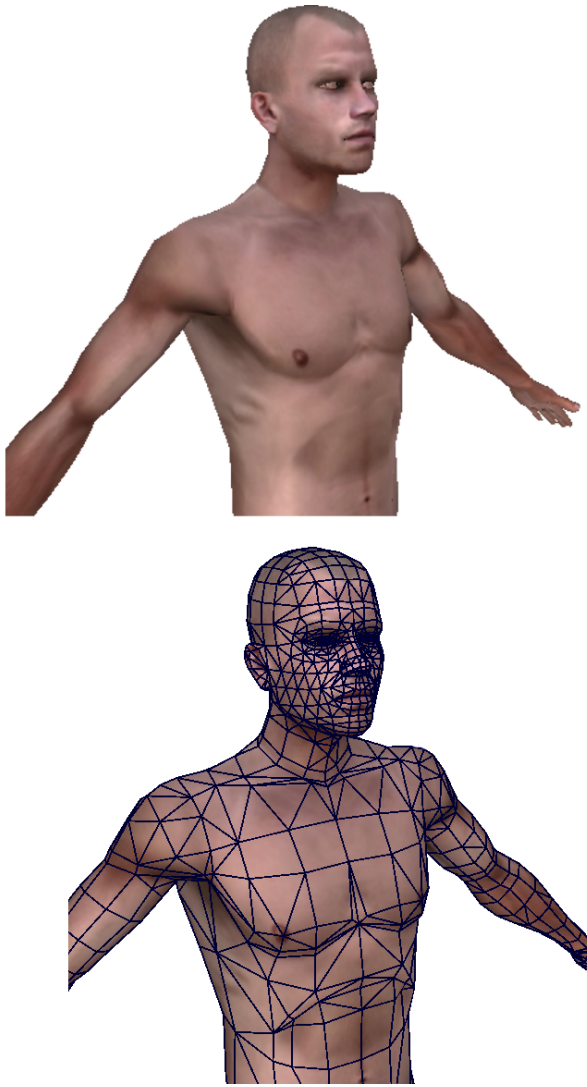
- Paramétrisation d'un maillage = Construction de S (par morceaux) étant donné Γ.



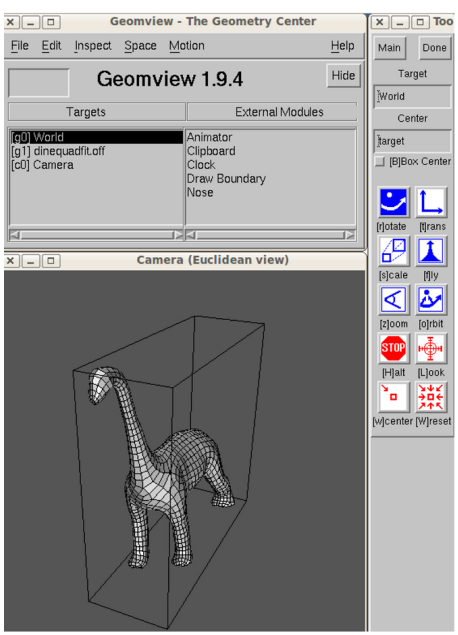
[Botsh, Pauly, Kobbelt, Alliez, Lévy, SIGGRAPH Course Notes 2007]

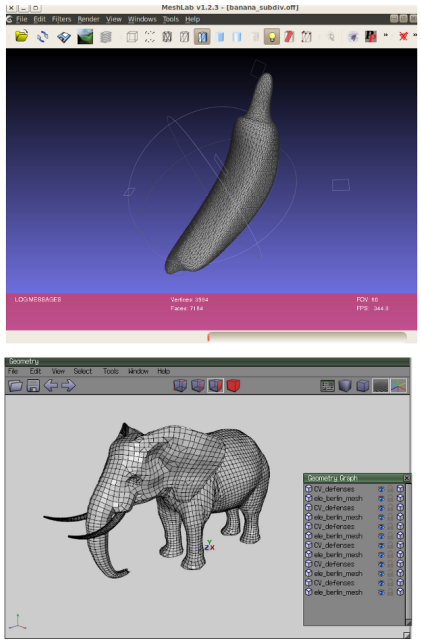
Textures

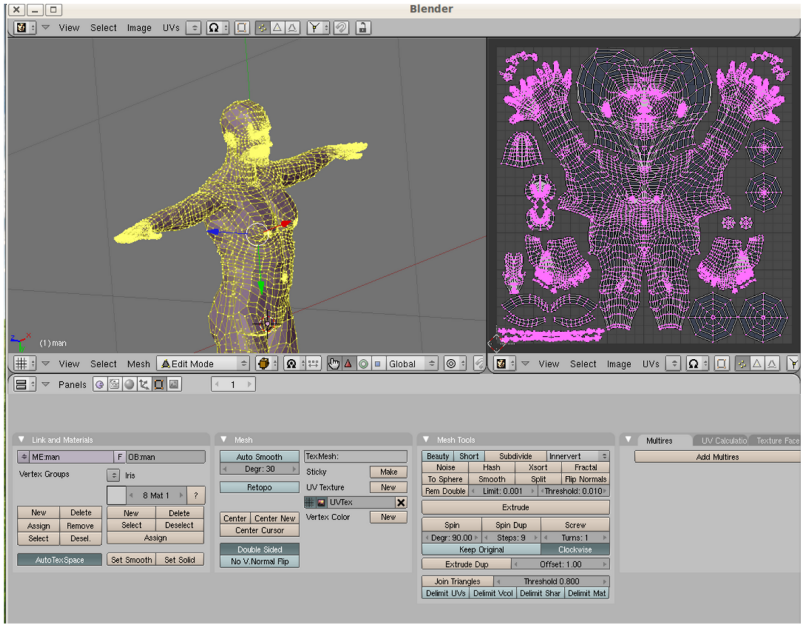
- Morceaux se recouvrants = atlas (charts).



Softwares







- Geomview (Viewer)
- Meshlab (Mesh Processing)
- Wings3D (Subdivision)
- Blender (Artiste)