

Le lancé de rayons: Du modèle 3D à l'image photoréaliste

001

Contexte

Rendu: **Modèle** de données **3D** → **Image**

Ex. Modèle 3D sphère:

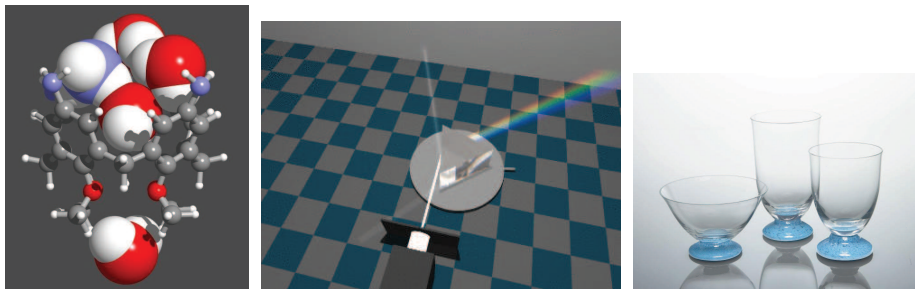
$$x^2 + y^2 + z^2 = 1 \rightarrow ?$$



002

Contexte

Exemple d'application:



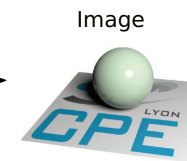
003

Contexte

Rendu: **Modèle** de données **3D** → **Image**

Ex. Modèle 3D sphère:

$$x^2 + y^2 + z^2 = 1 \rightarrow ?$$



Plusieurs solutions
(+/-)

- Rendu projectif
- Reye
- Lancé de rayons
(ray tracing)

004

Plan de la présentation

- 1/ Principe général du lancé de rayons
- 2/ Cas d'application sur une scène simple
=> *Voir des objets*
- 3/ L'illumination
=> *Aspect visuel plaisant*
- 4/ Modèle physique
=> *Vers le photoréalisme*

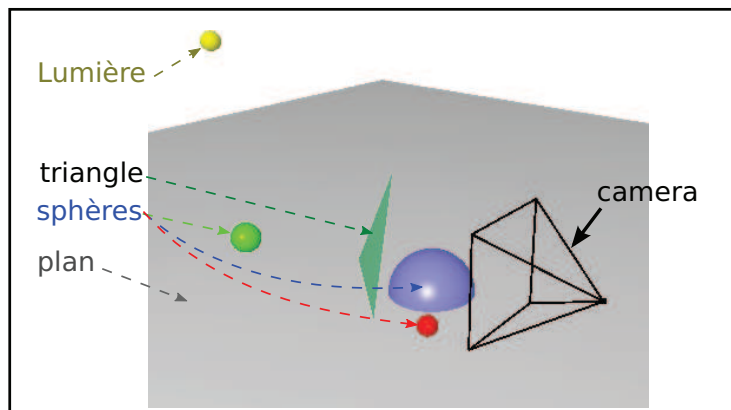
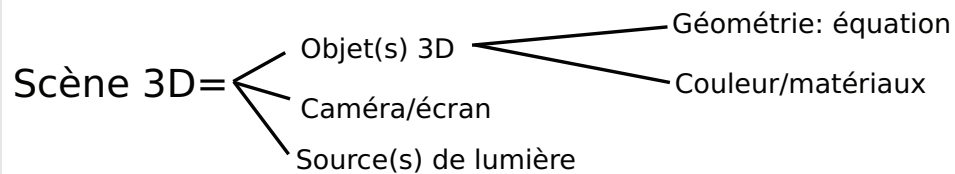
005

1/ Principe général du lancé de rayons

006

1/ Principe

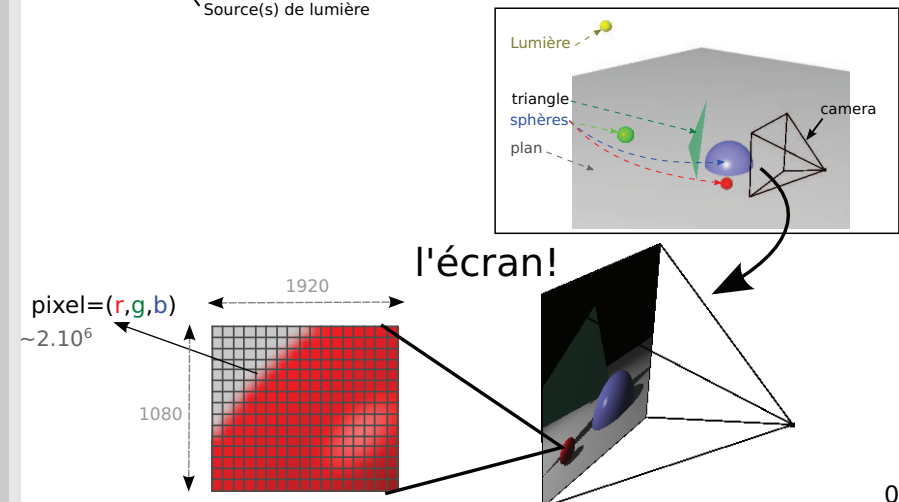
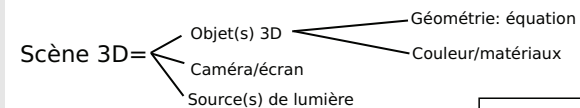
Contexte: Scène 3D



007

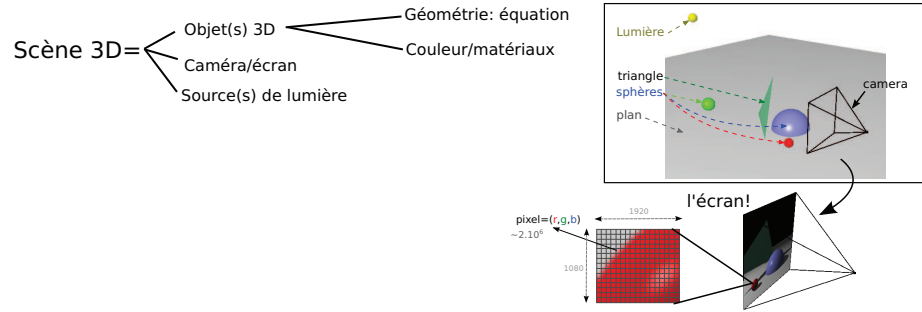
1/ Principe

Contexte: Scène 3D



008

Contexte: Scène 3D

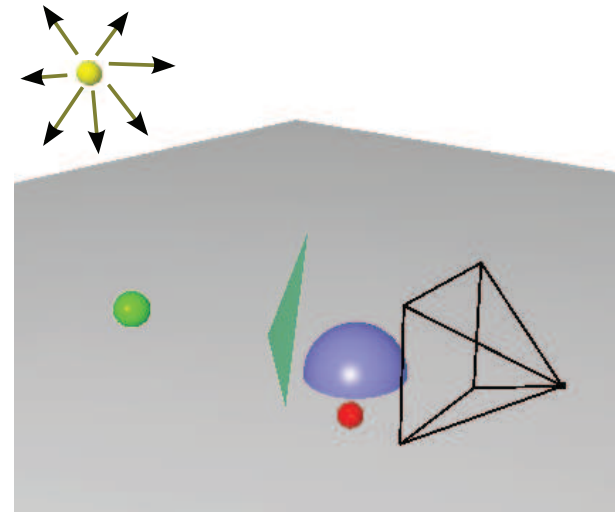


Question:

Quelles couleurs affecter aux pixels ?

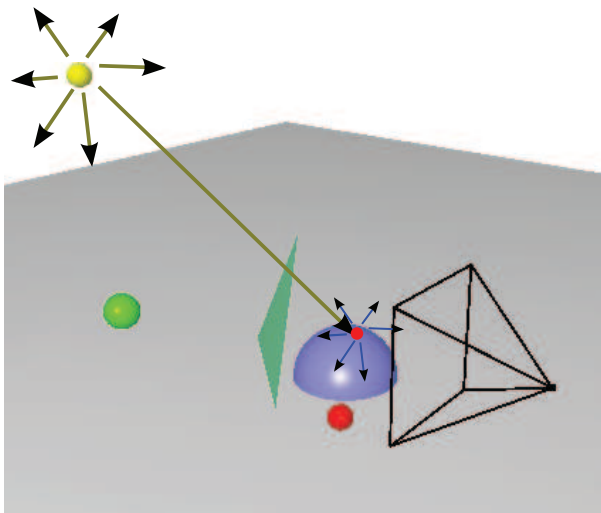
009

Dans la réalité?



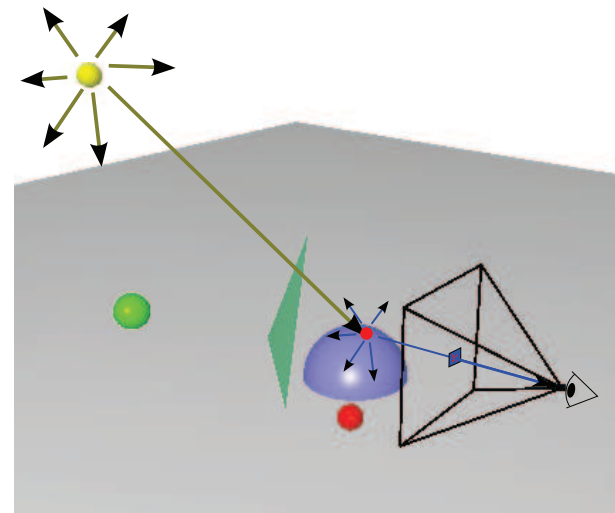
010

Dans la réalité?



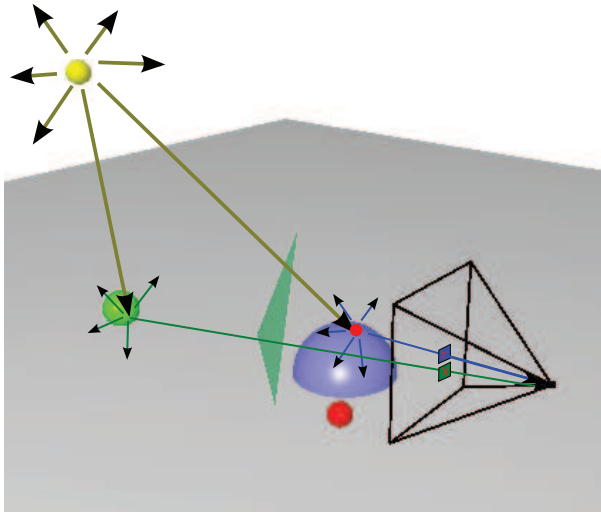
011

Dans la réalité?



012

Notre 1er modèle



```

Pour toutes sources lumiere
Pour toutes directions d1
  Lance_rayon(d1)
Fin Pour
Fin Pour

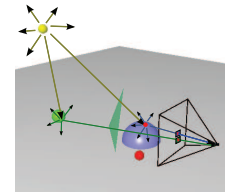
Lance_rayon(d)
{
Si d intersecte objet
  Sauve couleur
  Pour toutes directions d2
    Lance_rayon(d2)
  Fin Pour
Fin Si

Si d intersecte ecran
  Affecte couleur courante
Fin Si

Exit
}
  
```

013

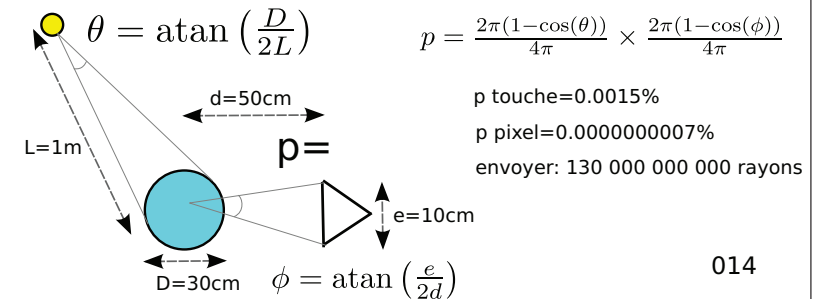
Notre 1er modèle



- + Conceptuellement simple
- + Physiquement ok
- Complexité algorithmique

↪ Recursions infinies

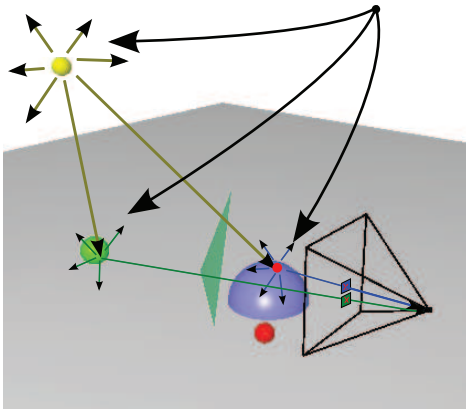
ex.



014

Acceleration

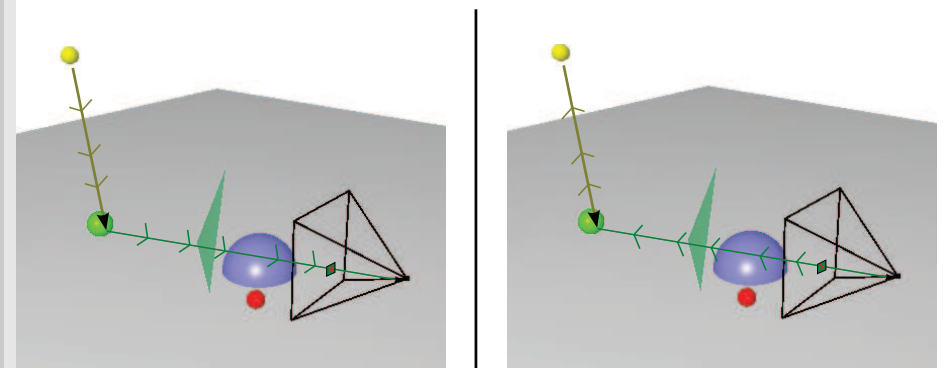
Problème = rayons inutiles



015

Acceleration

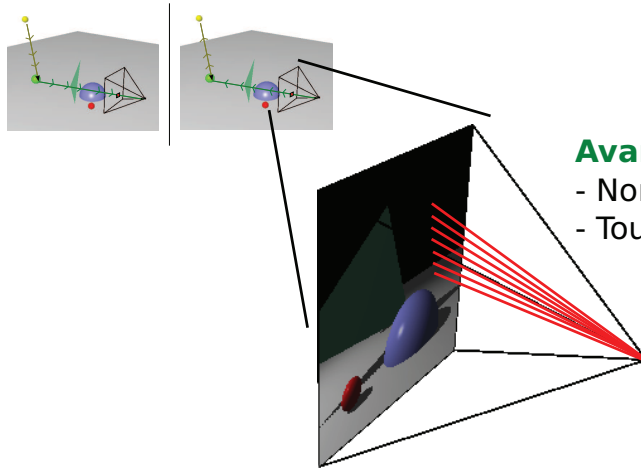
Principe du retour inverse de la lumière



016

Acceleration

Principe du retour inverse de la lumière



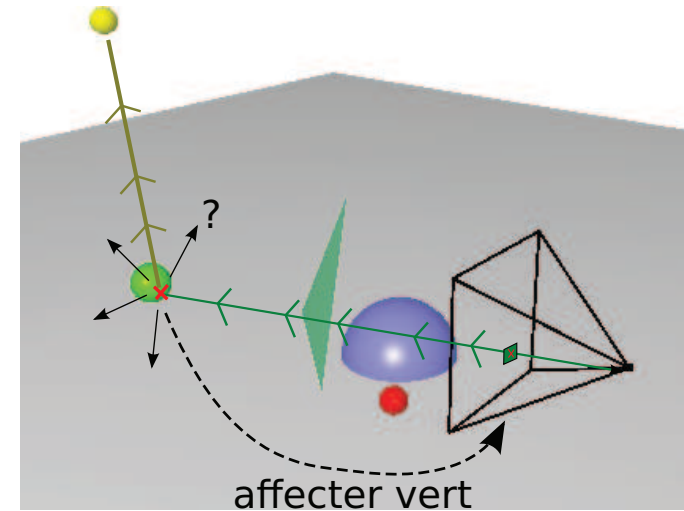
Avantage:

- Nombre fini de rayons
- Tous utiles

017

Acceleration

Principe du retour inverse de la lumière



018

Acceleration

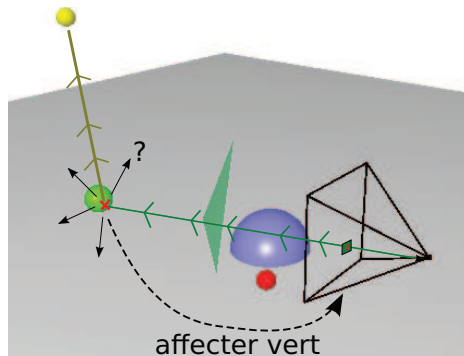
1/ Principe du retour inverse de la lumière
+ physique OK

2/ Pas de diffusion secondaire
- physique perdu

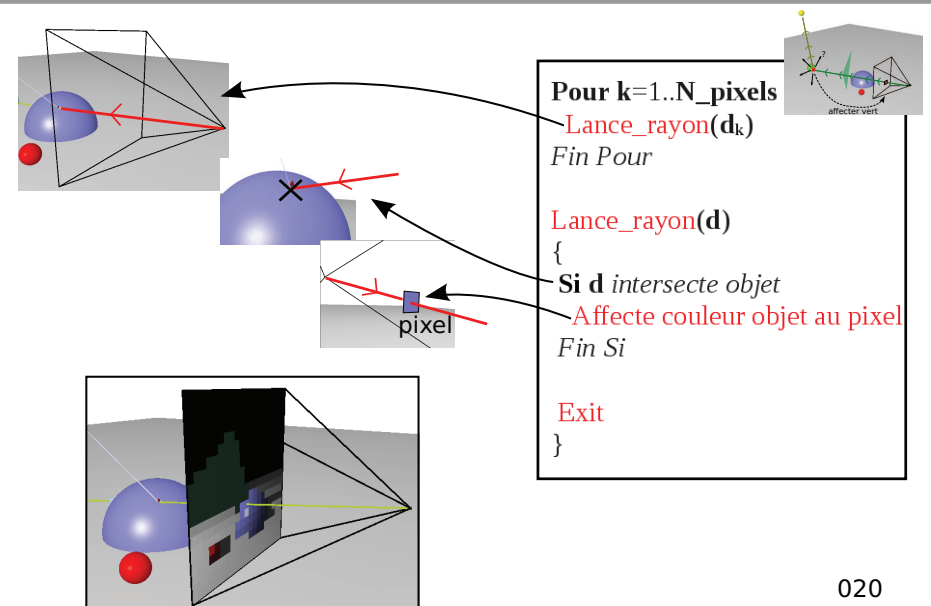
quantification:
~2 000 000 rayons

AN: 0.1ms/rayons :
3min/image
vs 150jours/image

019

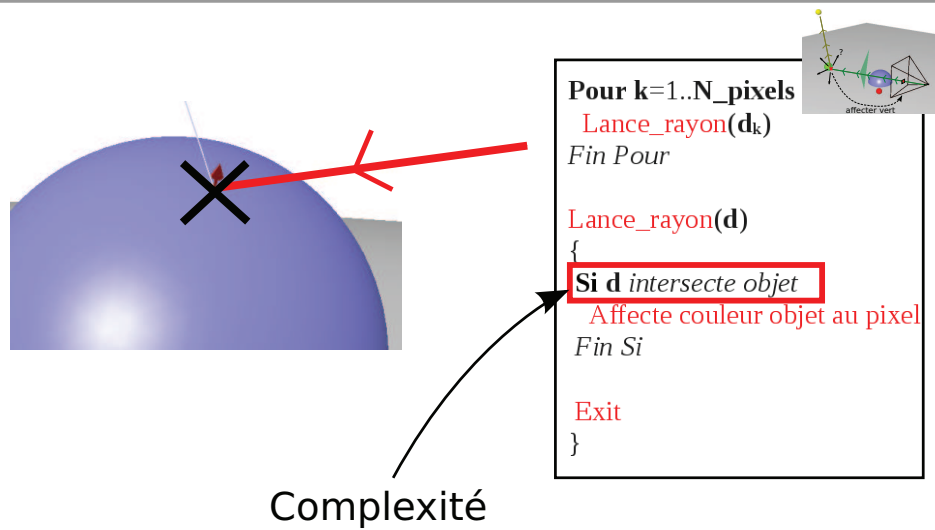


Recapitulatif



020

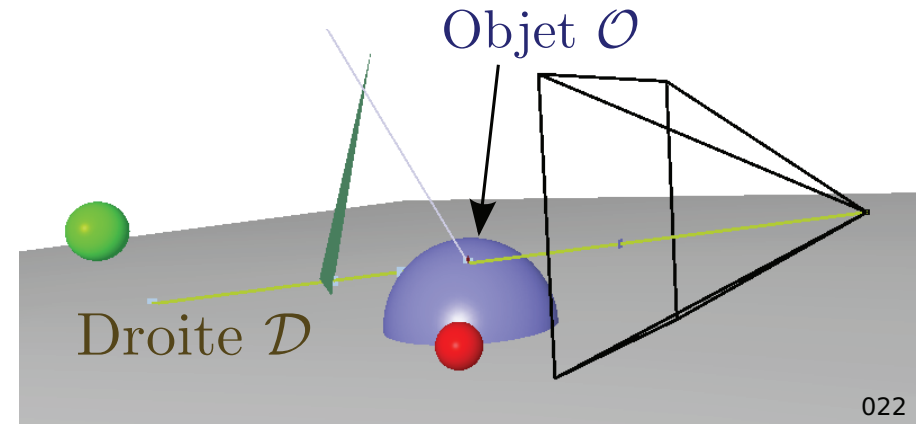
Recapitulatif



021

Formalisation

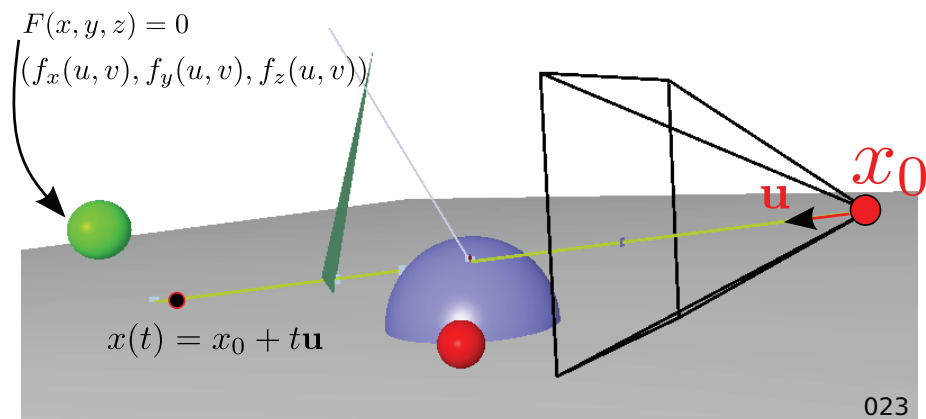
Pour toutes droites \mathcal{D}
 Connaitre $\mathcal{D} \cap \mathcal{O}$



022

Formalisation

Pour toutes droites \mathcal{D}
 Connaitre $\mathcal{D} \cap \mathcal{O}$

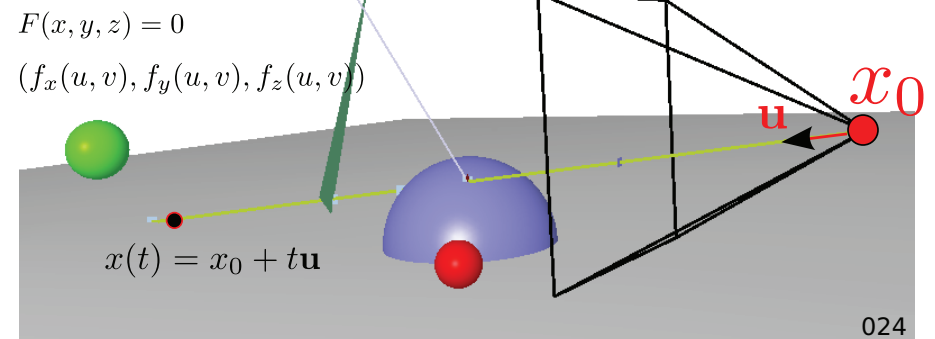


023

Formalisation

1: On cherche
 $F(x, y, z) = 0$
 $x(t) = x_0 + tu$

2: On résout
 pour t
 On en déduit:
 $x(t)$ et $n(t)$

On garde $t > 0$ Illumination
(propriétés locales)

024

2/ Cas d'application sur des scènes simples

025

Cas du plan

Il faut résoudre:

$$\begin{cases} x(t) = x_0 + t\mathbf{u} \\ \langle x(t) - x_p, \mathbf{n}_p \rangle = 0 \end{cases}$$

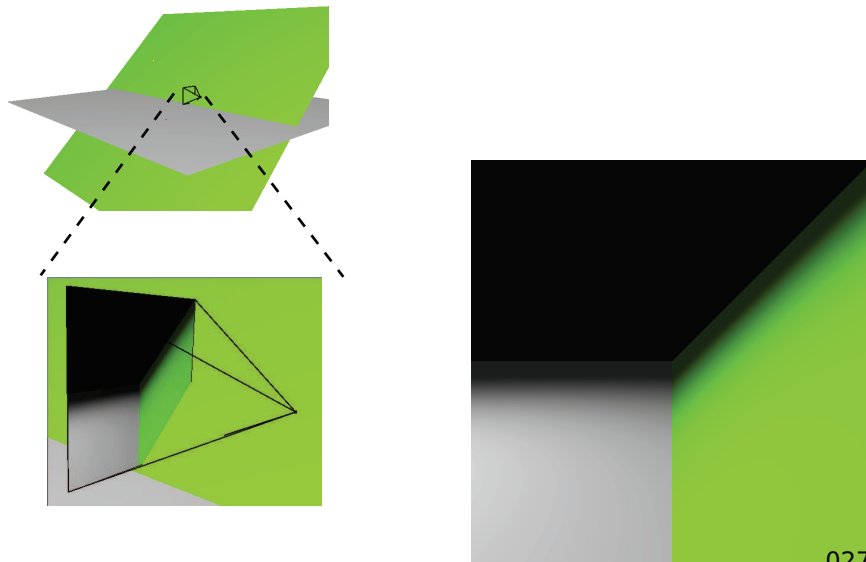
2 cas

- Coplanaire
- Intersection
 - $t > 0$ → 1 intersection
 - $t < 0$ → 0 intersection

$x \in \mathcal{P} \Rightarrow \langle x - x_p, \mathbf{n}_p \rangle = 0$

026

Cas du plan



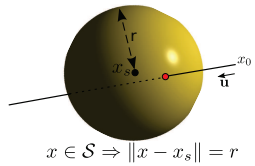
027

Cas de la sphère

$x \in \mathcal{S} \Rightarrow \|x - x_s\| = r$

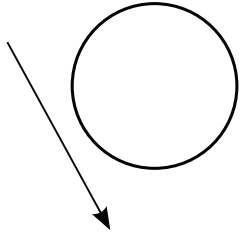
028

Cas de la sphère

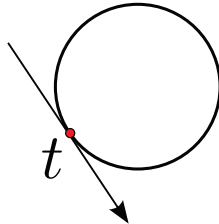


$$\begin{cases} x(t) = x_0 + tu \\ \|x(t) - x_s\| = r \end{cases}$$

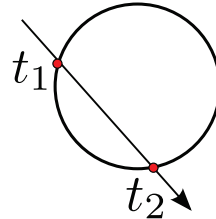
Cas 1:



Cas 2:

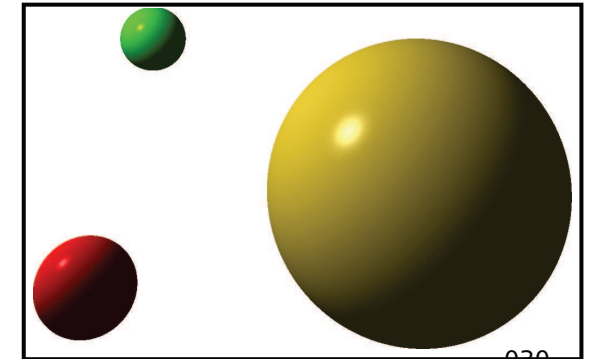
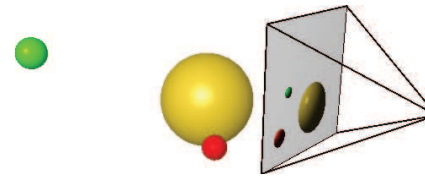


Cas 3:



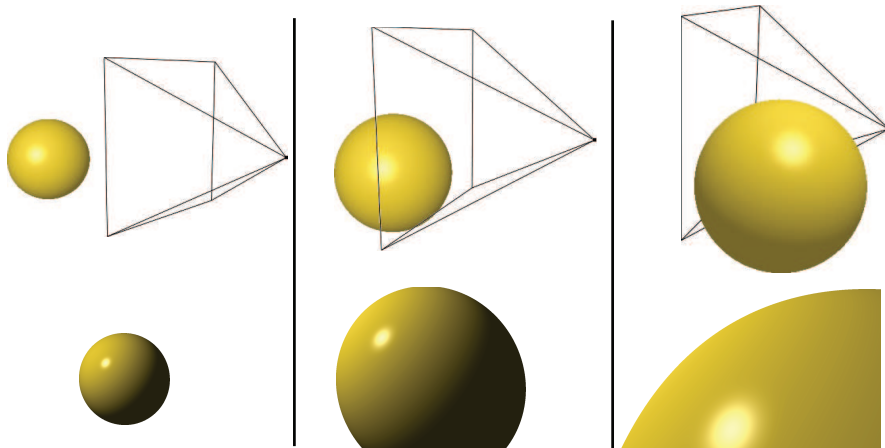
029

Cas de la sphère



Cas de la sphère

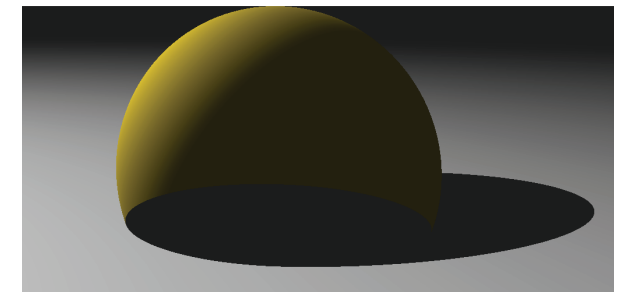
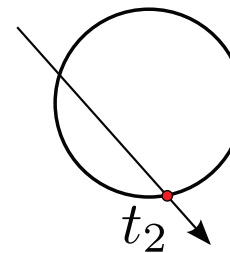
Avantage: Vraie sphère



031

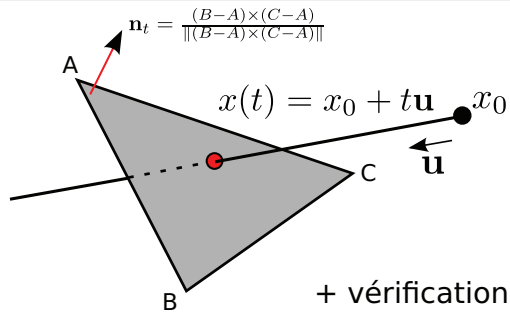
Cas de la sphère

Rem. Inversion d'intersection



032

Cas du triangle



Similaire plan

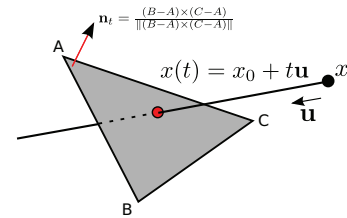
$$t = \frac{\langle A - x_0, \mathbf{n}_t \rangle}{\langle \mathbf{u}, \mathbf{n}_t \rangle}$$

+ vérification coordonnées barycentriques

$$x = \alpha A + \beta B + \gamma C$$

$$x \in \mathcal{T} \Rightarrow \begin{cases} \alpha + \beta + \gamma = 1 \\ 0 \leq \alpha \leq 1 \\ 0 \leq \beta \leq 1 \\ 0 \leq \gamma \leq 1 \end{cases}$$

Coord. barycentriques



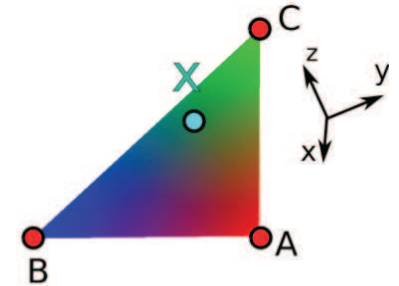
$$x = \alpha A + \beta B + \gamma C$$

$$\begin{cases} \alpha + \beta + \gamma = 1 \\ 0 \leq \alpha \leq 1 \\ 0 \leq \beta \leq 1 \\ 0 \leq \gamma \leq 1 \end{cases}$$

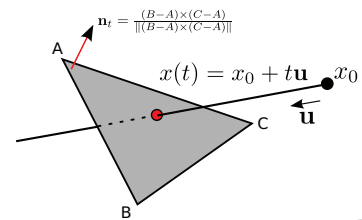
$$\begin{cases} A = \text{aire}(\mathbf{x}_B - \mathbf{x}_A, \mathbf{x}_C - \mathbf{x}_A) \\ A_1 = \text{aire}(\mathbf{x}_C - \mathbf{x}_B, \mathbf{x} - \mathbf{x}_B) \\ A_2 = \text{aire}(\mathbf{x}_A - \mathbf{x}_C, \mathbf{x} - \mathbf{x}_C) \\ A_3 = \text{aire}(\mathbf{x}_B - \mathbf{x}_A, \mathbf{x} - \mathbf{x}_A) \end{cases}$$

avec $\text{aire}(\mathbf{v}_0, \mathbf{v}_1) = 1/2 \|\mathbf{v}_0 \times \mathbf{v}_1\|$

$$\Rightarrow \begin{cases} \alpha = A_1/A \\ \beta = A_2/A \\ \gamma = A_3/A \end{cases}$$



Coord. barycentriques



$$t = \frac{\langle A - x_0, \mathbf{n}_t \rangle}{\langle \mathbf{u}, \mathbf{n}_t \rangle}$$

$$\begin{cases} \alpha + \beta + \gamma = 1 \\ 0 \leq \alpha \leq 1 \\ 0 \leq \beta \leq 1 \\ 0 \leq \gamma \leq 1 \end{cases}$$

```

v3 x10=internal_x1-internal_x0;
v3 x20=internal_x2-internal_x0;

v3 u10=x10.normalized();
v3 u20=x20.normalized();

v3 n=u10.cross(u20).normalized();

const v3 u=seg.u();
double proj=u.dot(n);

double epsilon=1e-8;
if(std::fabs(proj)<epsilon)
    return inter;

double t=(internal_x0-seg.x0()).dot(n)/proj;
v3 xi=seg(t);

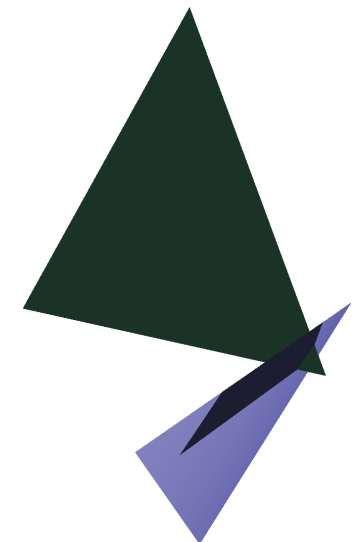
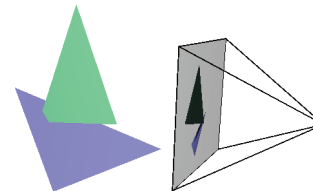
double area_0=(internal_x2-internal_x1).cross(xi-internal_x1).norm()/2.0;
double area_1=(internal_x0-internal_x2).cross(xi-internal_x2).norm()/2.0;
double area_2=(internal_x1-internal_x0).cross(xi-internal_x0).norm()/2.0;
double area =x10.cross(x20).norm()/2.0;

double a=area_0/area;
double b=area_1/area;
double c=area_2/area;

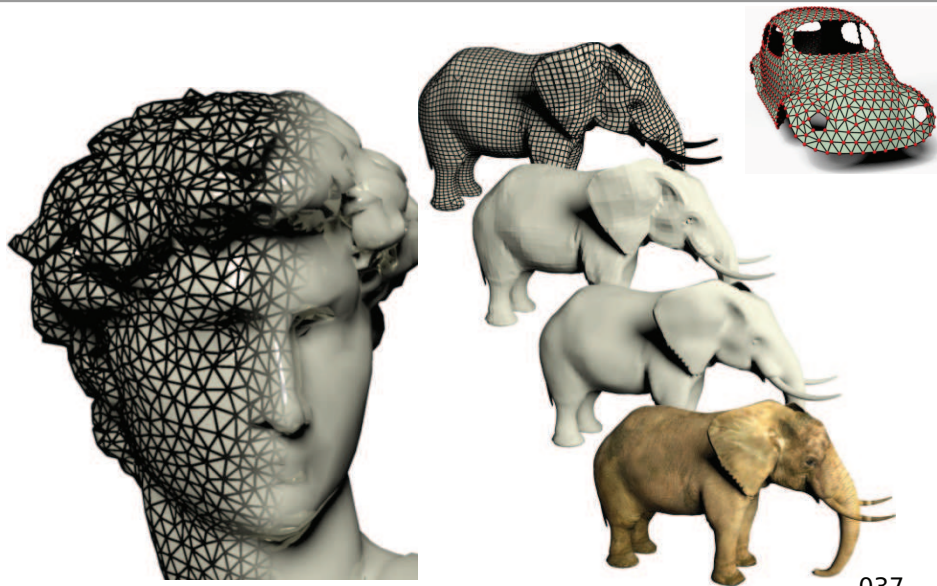
if(a==0 && b==0 && c==0 && a<1 && b<1 && c<1)
    if(std::fabs(a+b+c-1.0)<epsilon)
        inter.push_back(intersection_data(xi,n,t));

return inter;
    
```

Cas du triangle



Triangle => Maillage



037

Surfaces implicites

$$S = \{(x, y, z) \in \mathbb{R}^3 \mid F(x, y, z) = 0\}$$

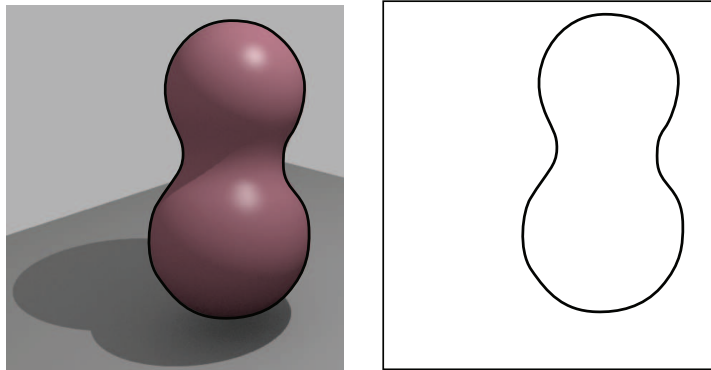
De manière générale, on ne connaît pas analytiquement $S \cap \mathcal{D}$

On recherche une approximation

038

Surfaces implicites

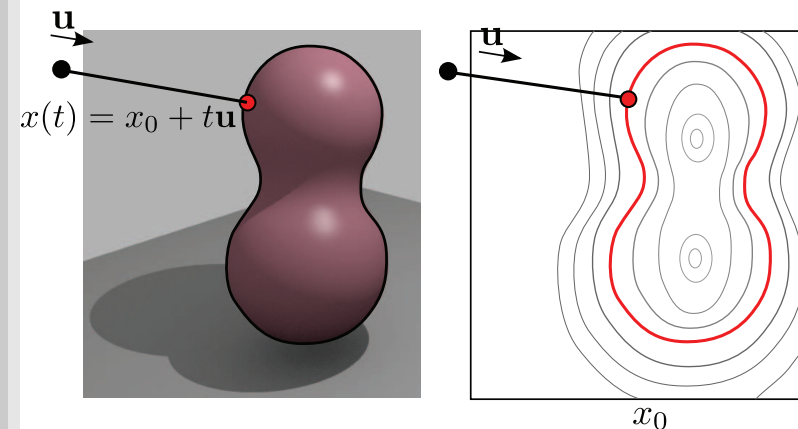
$$S = \{(x, y, z) \in \mathbb{R}^3 \mid F(x, y, z) = 0\}$$



039

Surfaces implicites

$$S = \{(x, y, z) \in \mathbb{R}^3 \mid F(x, y, z) = 0\}$$

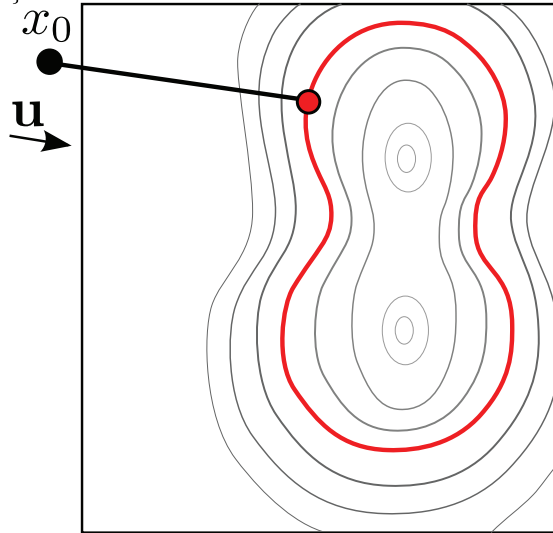
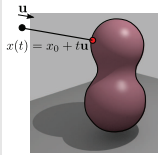


040

1/ Principe
2/ Application

Surfaces implicites

$$S = \{(x, y, z) \in \mathbb{R}^3 | F(x, y, z) = 0\}$$

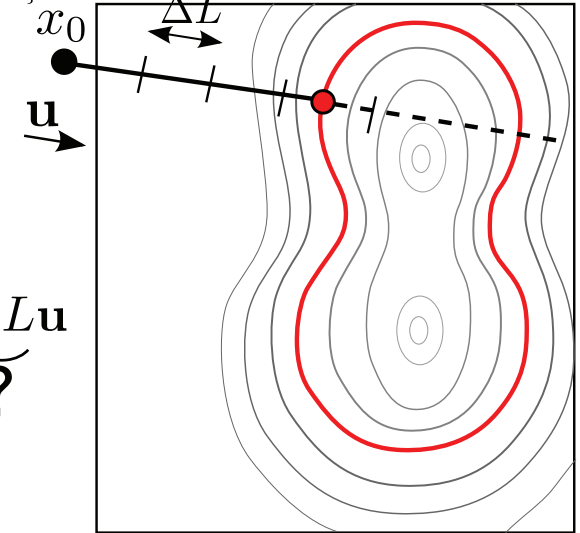
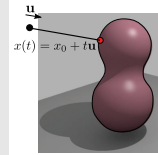


041

1/ Principe
2/ Application

Surfaces implicites

$$S = \{(x, y, z) \in \mathbb{R}^3 | F(x, y, z) = 0\}$$



Solution 1:

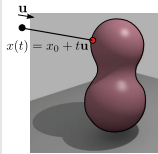
$$x^{i+1} = x^i + \underbrace{\Delta L}_{?} \mathbf{u}$$

042

1/ Principe
2/ Application

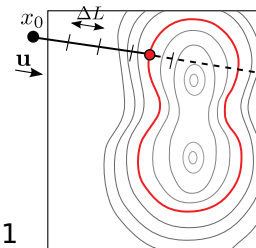
Surfaces implicites

$$S = \{(x, y, z) \in \mathbb{R}^3 | F(x, y, z) = 0\}$$



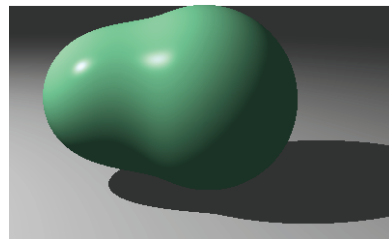
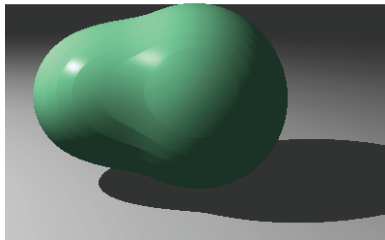
Solution 1:

$$x^{i+1} = x^i + \Delta L \mathbf{u}$$



0.2

0.001

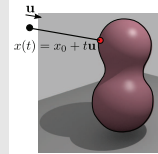


043

1/ Principe
2/ Application

Surfaces implicites

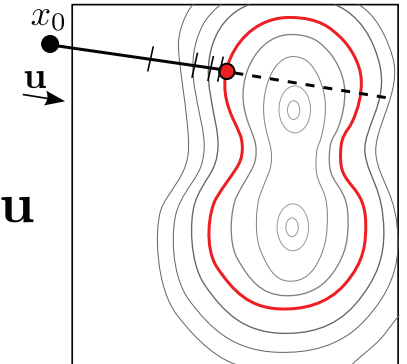
$$S = \{(x, y, z) \in \mathbb{R}^3 | F(x, y, z) = 0\}$$



F: dérivable

Solution 2:

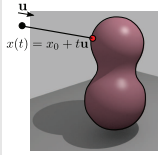
$$x^{i+1} = x^i + \frac{|F(x^i)|}{\|\nabla F\|_{\max}} \mathbf{u}$$



044

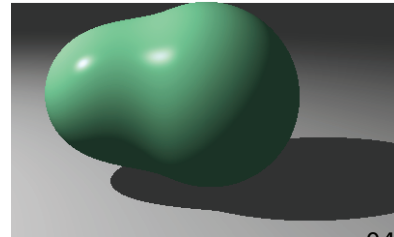
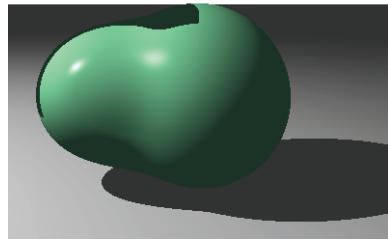
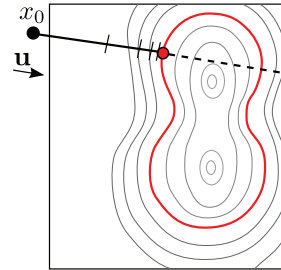
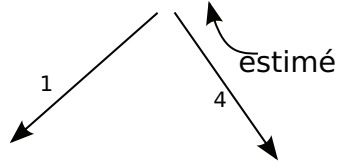
Surfaces implicites

$$S = \{(x, y, z) \in \mathbb{R}^3 | F(x, y, z) = 0\}$$



Solution 2:

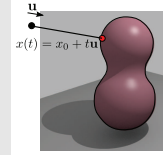
$$x^{i+1} = x^i + \frac{|F(x^i)|}{\|\nabla F\|_{\max}} \mathbf{u}$$



045

Surfaces implicites

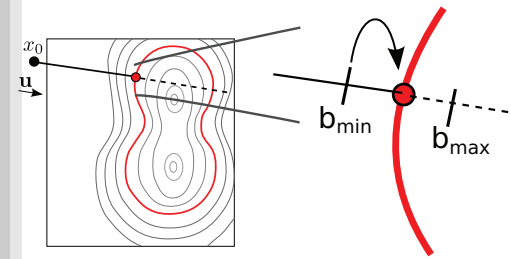
$$S = \{(x, y, z) \in \mathbb{R}^3 | F(x, y, z) = 0\}$$



Solution 2:

$$x^{i+1} = x^i + \frac{|F(x^i)|}{\|\nabla F\|_{\max}} \mathbf{u}$$

Acceleration convergence:



1: Dichotomie

2: Newton
à la fin

$$x^{i+1} = x^i - \frac{F(x^i)}{\langle \nabla F(x^i), \mathbf{u} \rangle}$$

quadratique

046

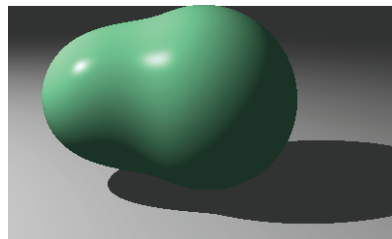
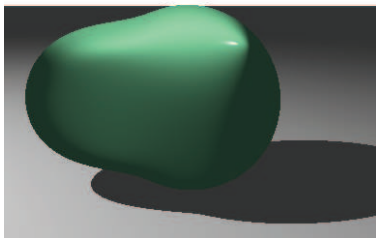
Surfaces implicites

$$S = \{(x, y, z) \in \mathbb{R}^3 | F(x, y, z) = 0\}$$

Importance du gradient:

$$\frac{F(x+h) - F(x)}{h}$$

∇F analytique



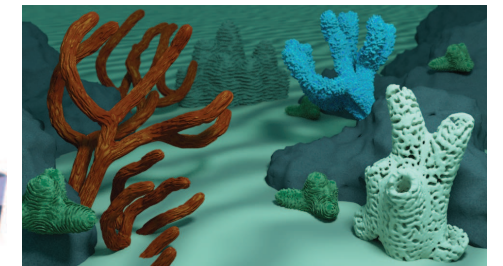
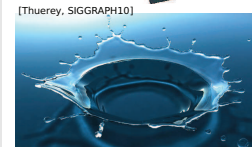
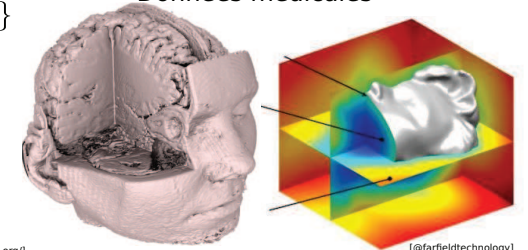
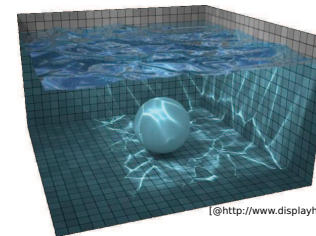
Attention à la convergence

047

Surfaces implicites

$$S = \{(x, y, z) \in \mathbb{R}^3 | F(x, y, z) = 0\}$$

Données médicales



Simulation de fluides

Modélisation

048

Illumination

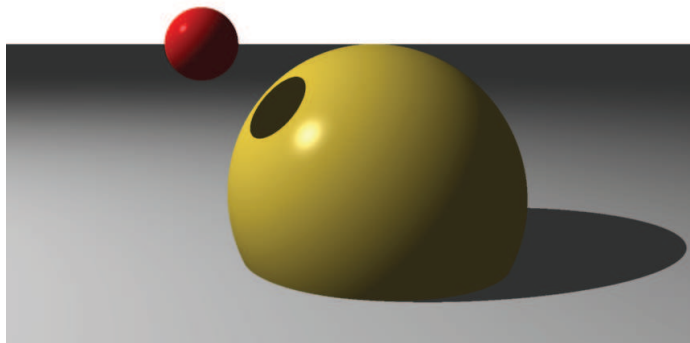
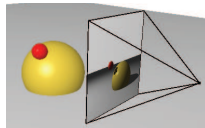
049

Effets visuels

050

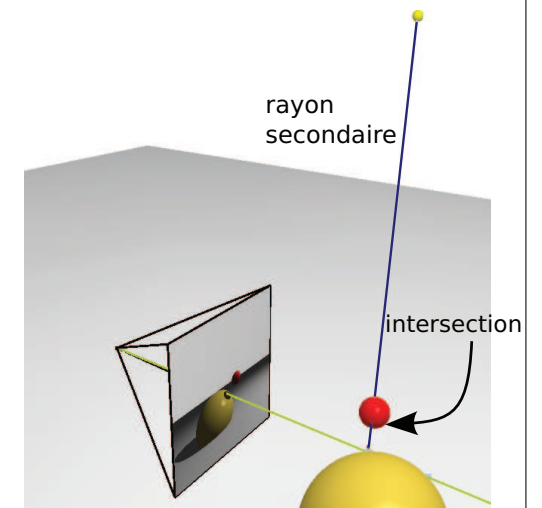
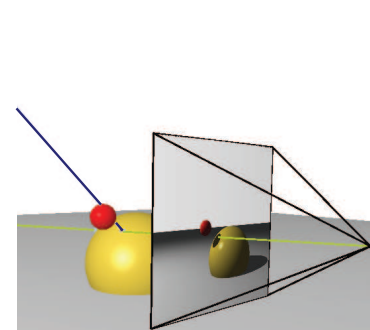
Ombres

Obtenues naturellement



051

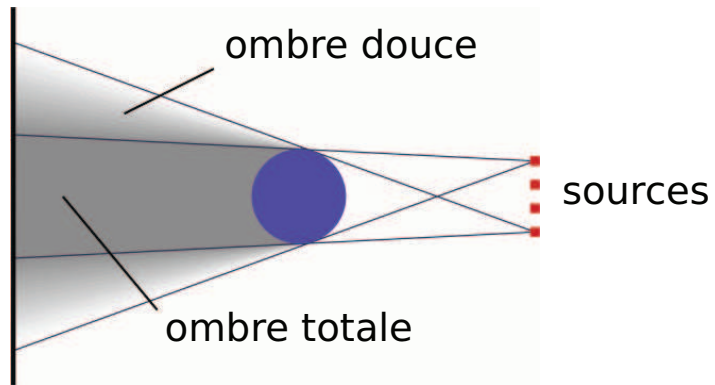
Ombres



052

- 1/ Principe
- 2/ Application
- 3/ Illumination

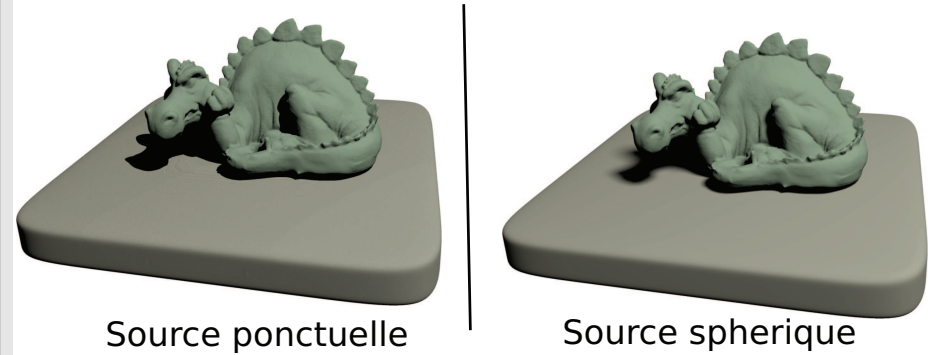
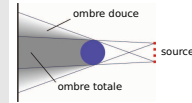
Ombres douces



053

- 1/ Principe
- 2/ Application
- 3/ Illumination

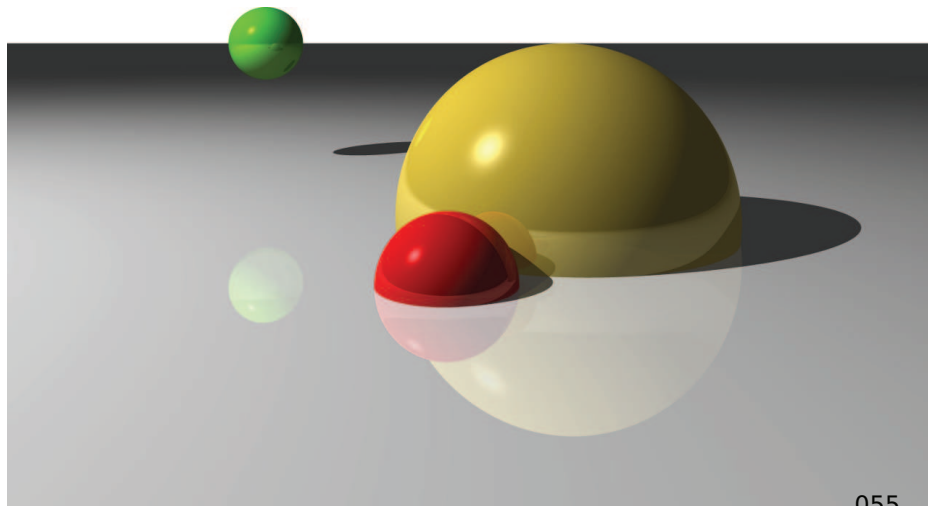
Ombres douces



054

- 1/ Principe
- 2/ Application
- 3/ Illumination

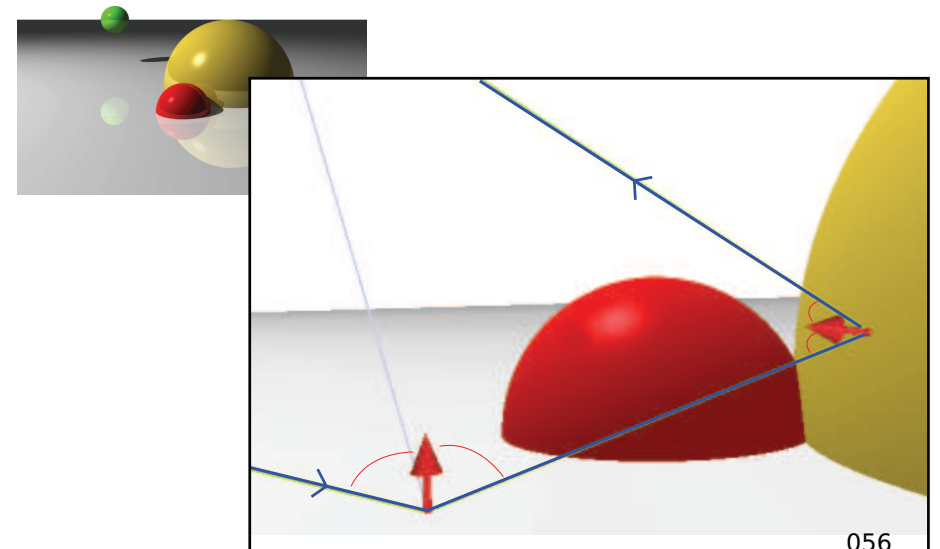
Reflexions



055

- 1/ Principe
- 2/ Application
- 3/ Illumination

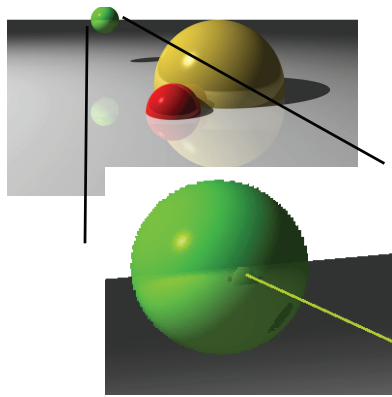
Reflexions



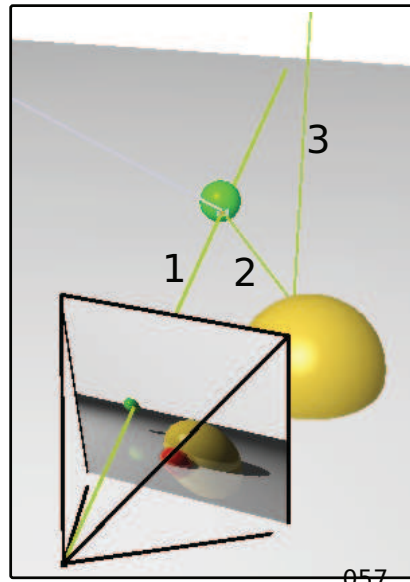
056

- 1/ Principe
- 2/ Application
- 3/ Illumination

Reflexions



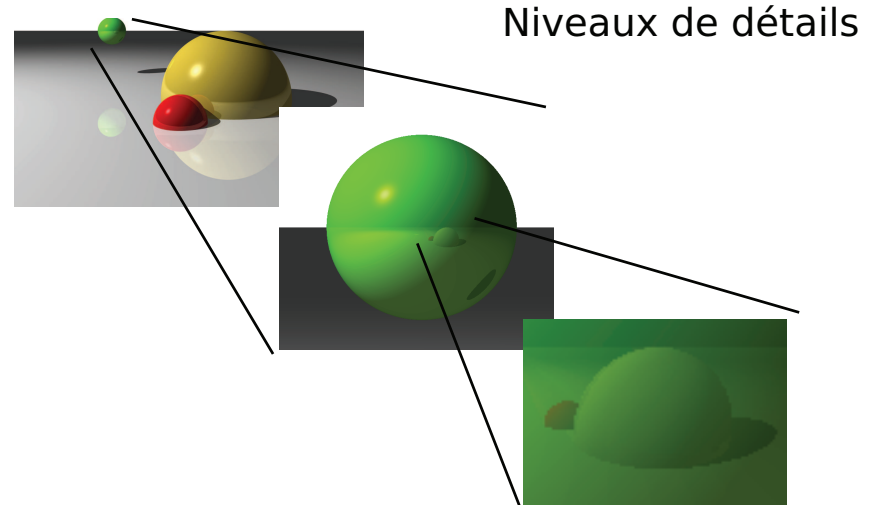
Plusieurs niveaux de réflexions:



057

- 1/ Principe
- 2/ Application
- 3/ Illumination

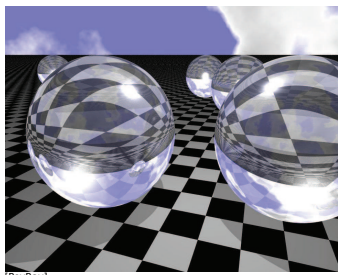
Reflexions



058

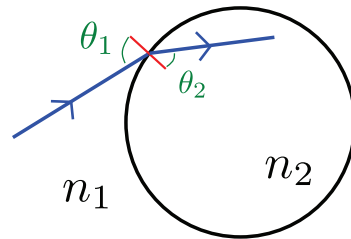
- 1/ Principe
- 2/ Application
- 3/ Illumination

Refraction



[PovRay]

$$n_1 \sin(\theta_1) = n_2 \sin(\theta_2)$$



059

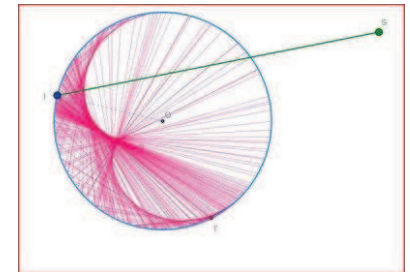
- 1/ Principe
- 2/ Application
- 3/ Illumination

Caustics

Trajet *privilegié* des rayons réfléchis/refractés



[http://abcmathsblog.blogspot.fr/]



060

- 1/ Principe
- 2/ Application
- 3/ Illumination

Caustics



[CG Arena]



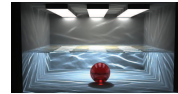
[deviantart]

N'est pas obtenu par le principe de base

061

- 1/ Principe
- 2/ Application
- 3/ Illumination

Caustics



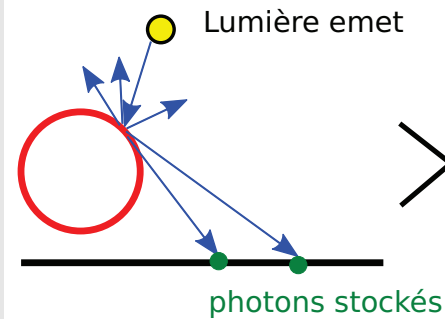
[CG Arena]



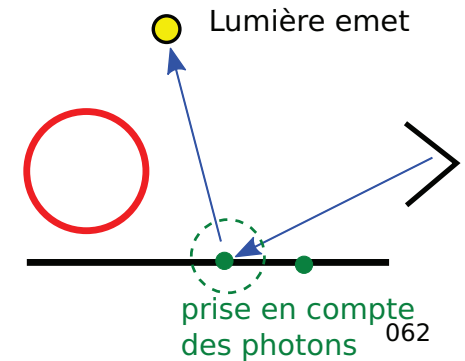
[deviantart]

Utilisation d'une *photon map*

Etape 1:
Lancé de *photons*



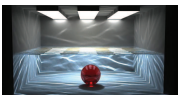
Etape 2:
Lancé de rayons



062

- 1/ Principe
- 2/ Application
- 3/ Illumination

Caustics

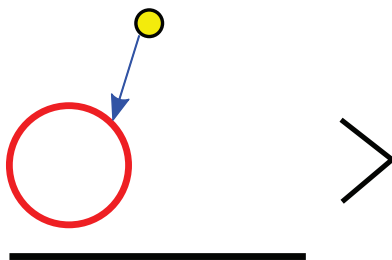


[CG Arena]



[deviantart]

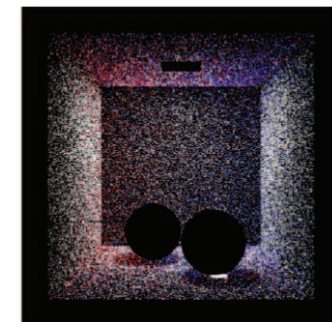
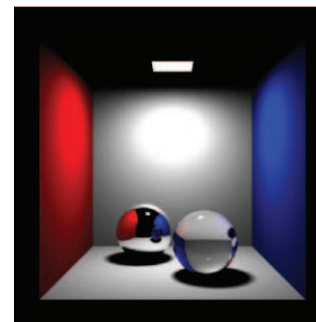
Utilisation d'une *photon map*



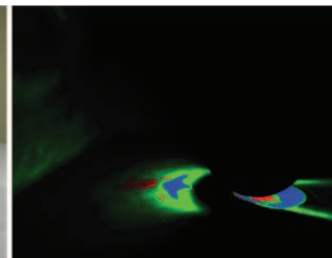
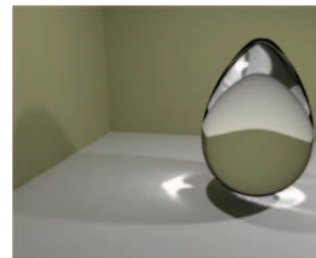
063

- 1/ Principe
- 2/ Application
- 3/ Illumination

Caustics



carte de photons



Quantification des caustics

[Jensen, EGWR96]

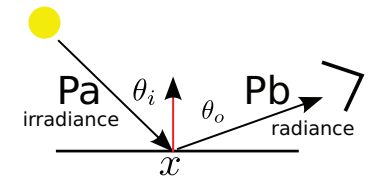
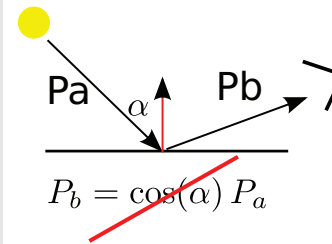
064

Modèle physique

Physically based rendering

BRDF

**Bidirectional
Reflectance
Distribution
Function**

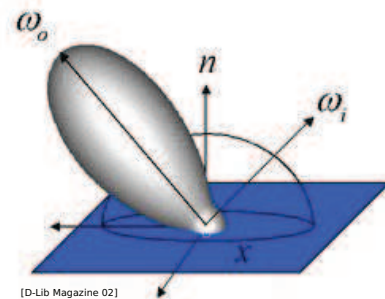
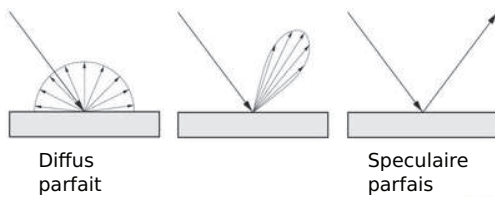


Si dépend de f => irridescence



BRDF

Bidirectional Reflectance Distribution Function



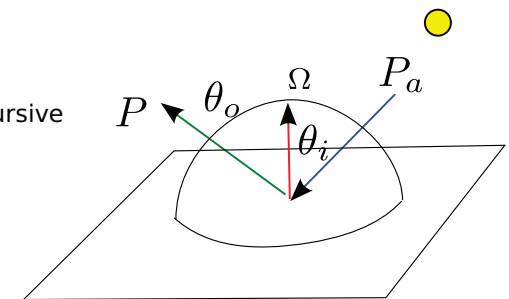
[D-Lib Magazine 02]

Equation de rendu

$$P(x, \theta_o) = P_e(x, \theta_o) + \int_{\theta_i \in \Omega} f(x, \theta_i, \theta_o) P_a(x, \theta_i) \cos(\theta_i) d\theta_i$$

Problème: P_a dépend de P

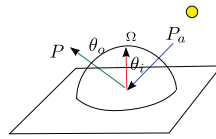
Equation integrale infiniment recursive



- 1/ Principe
- 2/ Application
- 3/ Illumination
- 4/ Physique

Equation de rendu

$$P(x, \theta_o) = P_e(x, \theta_o) + \int_{\theta_i \in \Omega} f(x, \theta_i, \theta_o) P_a(x, \theta_i) \cos(\theta_i) d\theta_i$$



2 Approches:

1- Discrétisation éléments finis

Radiosité

2- Monte-Carlo

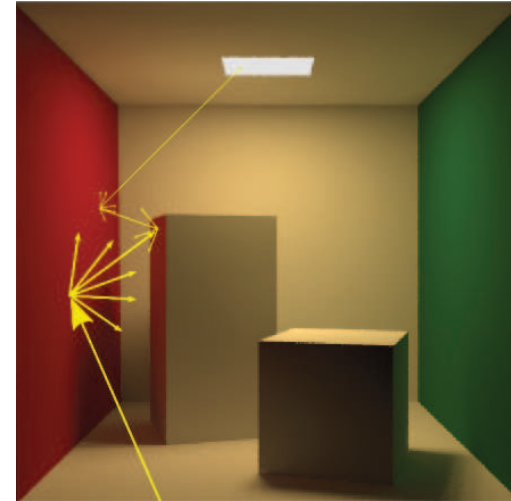
Path Tracing
Metropolis Light Transport (MTL)

069

- 1/ Principe
- 2/ Application
- 3/ Illumination
- 4/ Physique

Path tracing

On échantillonne θ_i au hasard



070

- 1/ Principe
- 2/ Application
- 3/ Illumination
- 4/ Physique

Path tracing

Modélisation sources secondaire:



Illumination directe



Path tracing

071