

ETI5 Majeure Image: Maillage avancé

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- Caractéristiques topologiques.
- Structure de données avancées : Halfedge.
- Subdivision de maillage.
- Notion de calcul sur variété : Lissage Laplacien.

Caractéristique topologique

Caractéristique d'Euler-Poincaré

- Maillage à N_f faces, N_s sommets, N_a arêtes.

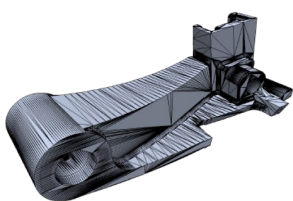
$$N_s - N_a + N_f = \chi = 2(c - g) - b$$

χ : Caractéristique d'Euler (Rappel : Gauss-Bonnet).

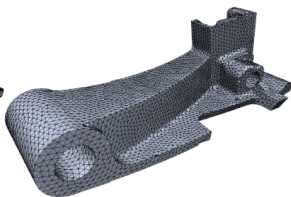
c : Nbr composante connexes

g : Nbr de trous (genre topologique)

b : Nbr de bords



Hetroy



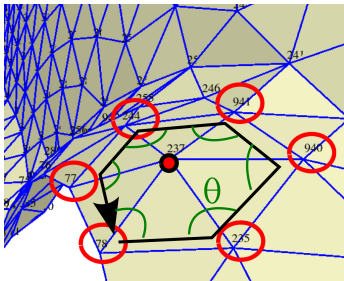
Wikipedia

Structure de données

Limitation encodage indexé

Encodage géométrie + connectivité indexée

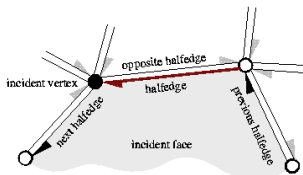
- + Affichage rapide (si contigu)
- + Générique
- + Simple
- Voisinage non encodé
- Ajout/suppression en $\mathcal{O}(N)$.



```
OFF
40 95 75
-0.175114 -0.047799 -0.045492
-0.199566 0.730914 -0.064795
-0.010689 0.674496 0.008900
-0.015538 0.153071 0.107408
-0.070148 0.767894 -0.116107
-0.053836 -0.792815 0.109714
-0.162416 -0.785481 0.088014
-0.112365 -0.792492 0.135482
-0.240928 0.031451 0.031966
-0.259289 0.209557 0.035420
0.296891 -0.707385 0.143375
-0.190129 -0.069002 0.109358
-0.010148 0.024179 -0.067283
-0.112968 -0.009127 0.092391
-0.185828 0.377372 -0.111155
3 20 4 1
3 34 11 13
3 12 30 0
3 30 13 17
3 23 22 21
3 29 38 17
3 32 0 13
3 14 0 37
3 24 4 21
3 14 32 1
3 24 2 22
3 3 12 25
3 4 24 15
3 21 15 26
3 35 34 13
3 19 32 13
3 19 13 27
```

Encodage Halfedge

- Encodage Demi-Arêtes (Halfedge)
- Encodage des arêtes : On retrouve les faces par parcours (variété).
- Ajout/suppression en $\mathcal{O}(1)$.



Vertex
Halfedge_handle halfedge()

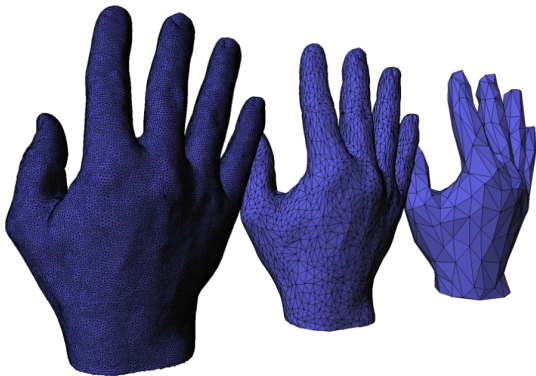
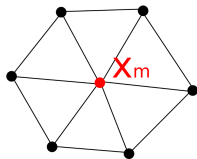
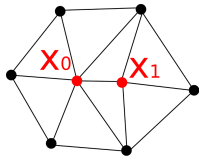
Halfedge
Halfedge_handle opposite()
Halfedge_handle next()
Halfedge_handle prev()
Vertex_handle vertex()
Face_handle face()

Face
Halfedge_handle halfedge()

CGAL

Edge Collapse

- Suppression d'arête (edge collapse) = Base de la simplification de maillage



CGAL

Encodage Halfedge comparaison

Choisir la structure la plus adaptée :

- Tableaux d'indices (vecteurs contigus)

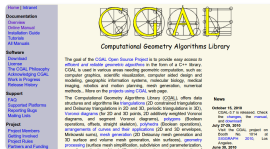
- + Simple, général, adapté GPU.
- + Accès aléatoire $\mathcal{O}(1)$.
- Parcours voisinage en $\mathcal{O}(N)$.
- Ajout/Suppression $\mathcal{O}(N)$.

- Halfedge

- + Parcours voisinage en $\mathcal{O}(1)$.
- + Ajout/Suppression en $\mathcal{O}(1)$.
- Technique, variété uniquement, encodage non contigu.
- Accès aléatoire en $\mathcal{O}(N)$.

```
Halfedge_handle g = h->next()->opposite()->next();
P.split_edge( h->next());
P.split_edge( g->next());
P.split_edge( g);
h->next()->vertex()->point() = Point( 1, 0, 1);
g->next()->vertex()->point() = Point( 0, 1, 1);
g->opposite()->vertex()->point() = Point( 1, 1, 0);
Halfedge_handle f = P.split_facet( g->next(),
                                   g->next()->next()->next());
Halfedge_handle e = P.split_edge( f);
e->vertex()->point() = Point( 1, 1, 1);
P.split_facet( e, f->next()->next());
```

Libraries implémentant le Halfedge



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The CGAL Philosophy
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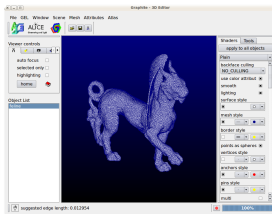
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Computational Geometry Algorithms Library

The goal of the CGAL, Open Source Project is to provide easy access to efficient and reliable geometric algorithms in the form of a C++ library. CGAL is used in various areas ranging from computer graphics, such as computer graphics, scientific visualization, computer aided design and modeling, geographic information systems, molecular biology, medical imaging, robotics and motion planning, mesh generation, hierarchical methods. More on the project using CGAL web page.

News
October 16, 2009
CGAL 3.10 released. Check the changes on the [CGAL website](#).
July 20th, 2010
The new CGAL impact on [CGAL 3.10](#).
August 16, 2010
CGAL 3.10.1 released.
June 26, 2010

introduction page.' Below this, there is a 'News' section with the text 'Monday 08. of March 2010 OpenMesh 2.0-RC5' and 'Download 2.0-RC5 for Linux or Windows. The download is available.' At the bottom, there is a footer with the text 'Copyright © 2009-2010 OpenMesh. All rights reserved.'." data-bbox="653 261 913 488"/>

Computer Sciences 8
Computer Graphics & Multimedia
Prof. Dr. Leif Kobbelt

RWTH AACHEN UNIVERSITY

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OpenMesh.org

Welcome to the OpenMesh website!

OpenMesh is a generic and efficient data structure for representing and manipulating polygonal meshes. For more information about OpenMesh and its features take a look at the [introduction page](#).

On top of OpenMesh we develop a flexible primary modeling and processing framework based on OpenMesh. For more details visit the [OpenFlipper website](#).

News

Monday 08. of March 2010 OpenMesh 2.0-RC5

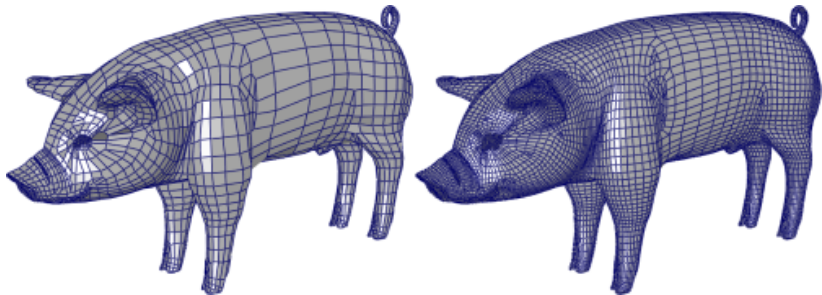
Download 2.0-RC5 for Linux or Windows. The download is available.

- CGAL (C++ complexe, calcul exact, beaucoup d'algos)
- Graphite (remaillage, paramétrisation, GUI)
- OpenMesh (plus simple que CGAL, moins complet)

Subdivision

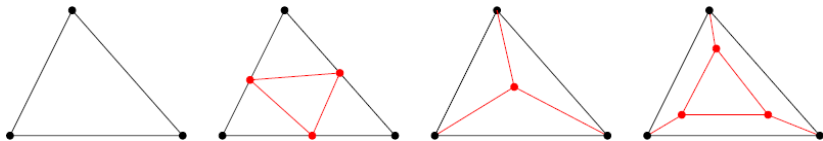
Subdivision de maillage

- Raffiner pour : rendu, calcul, déformer, . . .
- Voir cours SIGGRAPH



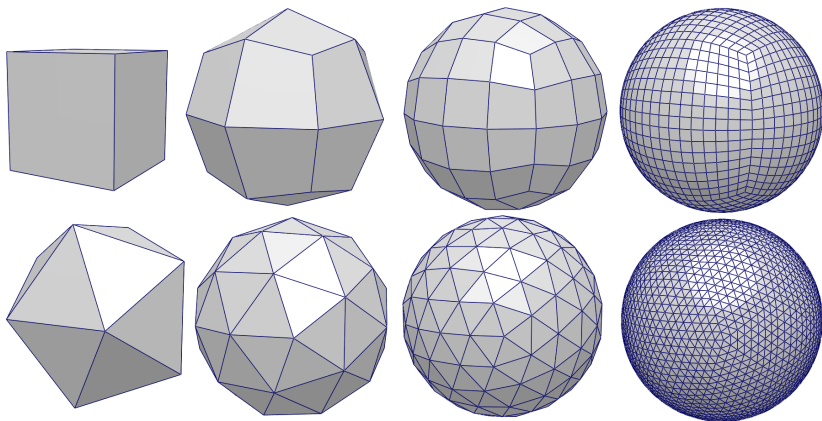
Subdivision de maillage

- Subdivision de la connectivité.
- Plusieurs possibilités de subdivision.



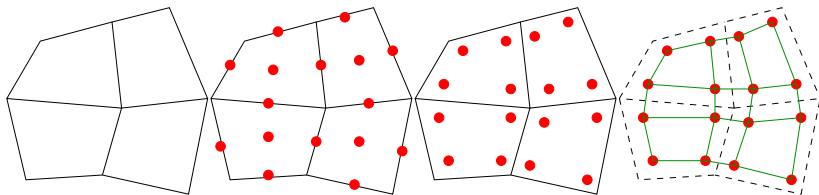
- Présences de polygones quelconques ...
- Structure de données !
- Géométrie
 - Schémas interpolants
 - Schémas approximants

- Application à la génération d'une sphère par projection :



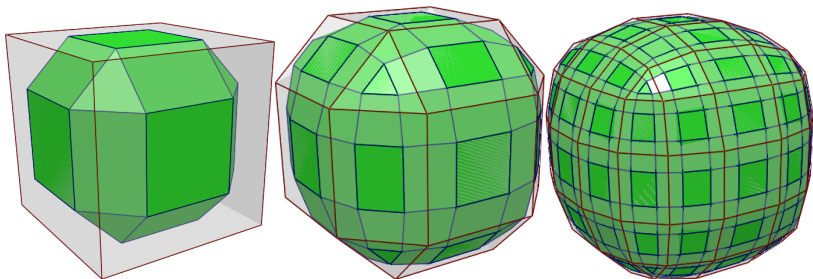
Exemple : Doo-Sabin

- 1 Soit une face formée par $(x_i)_{i=\llbracket 0, N-1 \rrbracket}$.
- 2 Construit sommets milieux $m_i = (x_i + x_{i+1})/2$.
- 3 Barycentre de la face $b = (\sum_i x_i)/N$.
- 4 Nouveaux sommets = $n_i = (x_i + m_i + m_{i-1} + b)/4$.



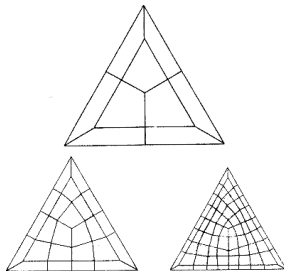
Subdivision

Exemple : Doo-Sabin



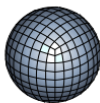
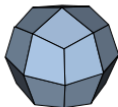
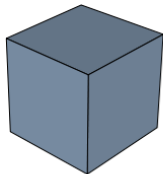
- Un sommet de face = barycentre de l'ancienne face
- Un sommet d'arête = moyenne entre le milieu des anciens sommets et le milieu des faces partageant l'arête
- Nouvelle position du sommet = $\frac{Q+2R+S(n-3)}{n}$

- Q : Moyenne des sommets de face adjacents
- R : Moyenne des points milieux des arêtes incidentes
- S : Anciennes coordonnées
- n : valence



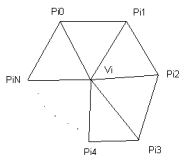
Catmull-Clark Resultats

- C^2 sauf au sommets extra-ordinaires
- Schema d'approximation
- Subdivision de face
- Basé de préférence sur des quads



Maillage Triangulaire

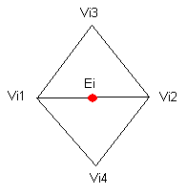
■ Nouvelle Position



$$V^{i+1} = (1 - n\alpha)V^i + \alpha \sum_{k=0}^n P_k$$

$$\alpha = \frac{1}{n} \left(\frac{5}{8} - \left(\frac{3}{8} - \frac{1}{4} \cos \left(\frac{2\pi}{n} \right) \right)^2 \right)$$

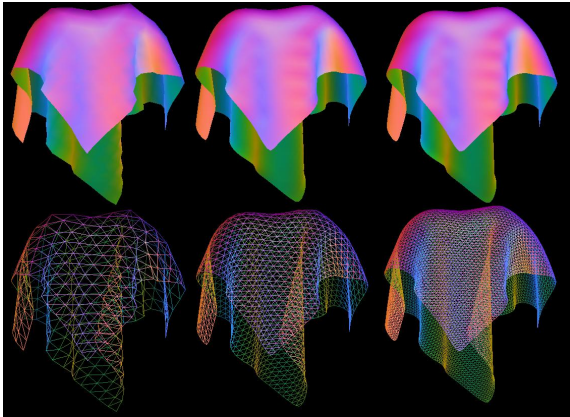
■ Sommets d'arêtes



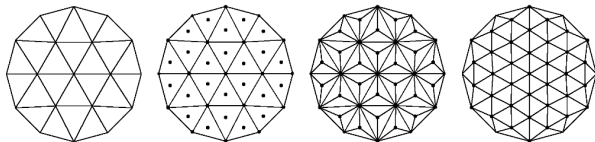
$$E^{i+1} = \frac{3}{8}(V_1 + V_2) + \frac{1}{8}(V_3 + V_4)$$

Loop

- C^2 sauf au sommets extra-ordinaires
- Schema d'approximation
- Subdivision de face
- Subdivision de triangles



Maillage Triangulaire

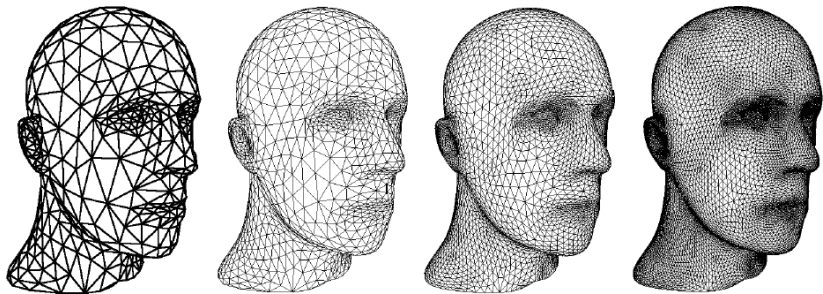


- Nouveaux sommets : barycentre de l'ancienne face
- Nouvelle position du sommet i :

$$(1 - \alpha_n)p_i + \frac{\alpha_n}{n} \sum_{j \in \mathcal{V}_i} p_j \quad (n=\text{valence}, \mathcal{V}=\text{voisinage})$$

$$\text{et } \alpha_n = \frac{4 - 2 \cos\left(\frac{2\pi}{n}\right)}{9}$$

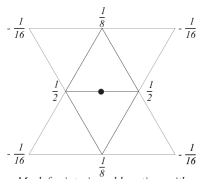
- C^2 sauf au sommets extra-ordinaires
- Schema d'approximation
- Subdivision de face
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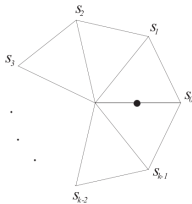
[Kobbelt, SIGGRAPH 00]

Maillage Triangulaire

■ Sommet régulier



■ Sommet non régulier



■ Ajout d'un point par coté

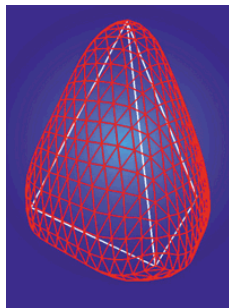
■ Dans le cas d'un sommet non régulier :

$$s_j = \frac{1}{k} \left(\frac{1}{4} + \cos \left(\frac{2\pi}{k} \right) + \frac{1}{2} \cos \left(\frac{4\pi}{k} \right) \right) \quad \text{pour } k > 5$$

$$s_0 = \frac{5}{12}, s_{1,2} = -\frac{1}{12} \quad \text{pour } k = 3$$

$$s_0 = \frac{3}{8}, s_2 = -\frac{1}{8}, s_{1,3} = 0 \quad \text{pour } k = 4.$$

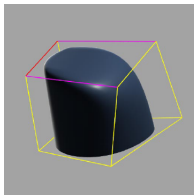
- C^1 sauf au sommets extra-ordinaires
- Schema d'interpolation
- Subdivision de face
- Subdivision de triangles



Subdivision : Arêtes vives

- Ne pas lisser toutes les arêtes :

[De Rose, Kass, Truong.
**Subdivision Surfaces in
Character Animation.** *ACM
SIGGRAPH.* 1998]

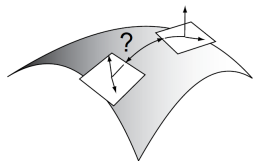


Lissage Laplacien

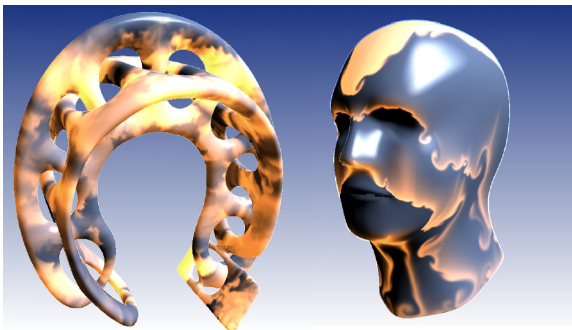
Lissage Laplacien

- Soit f définie sur une variété différentielle (surface lisse).

$$f : \begin{cases} \Gamma & \rightarrow \mathbb{R}^N \\ (\xi_1, \xi_2) & \mapsto f(\xi_1, \xi_2) \end{cases}$$



- Calcul différentiel ?
- Problème : Comparer vecteurs dans différents espaces



- Opérateur de Laplace-Beltrami = Laplacien sur une variété

$$\Delta = \frac{1}{\sqrt{\det(I_\Gamma)}} \sum_i \frac{\partial}{\partial \xi_i} \left(\sqrt{\det(I_\Gamma)} \sum_j I_\Gamma^{ij} \frac{\partial}{\partial \xi_j} \right)$$

- Cas particulier : Laplacien des propres coordonnées !
 $f = \mathbf{x}(\xi_1, \xi_2) = (x(\xi_1, \xi_2), y(\xi_1, \xi_2), z(\xi_1, \xi_2))$
- $\text{Sp}(\Delta \mathbf{x}) = \mathbf{modes\ de\ vibrations\ propres} = \text{Base de Fourier}$
Théorie Spectrale



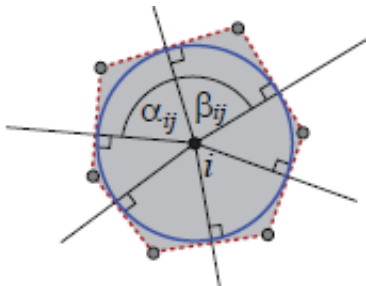
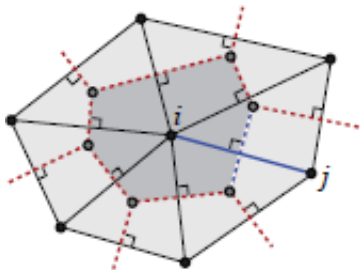
Lissage Laplacien

- Filtre passe bas = Convolution par filtre Gaussien en 2D = Solution de l'équation de diffusion (voir *Scale Space theory*).

$$\frac{\partial x}{\partial t} = \Delta \mathbf{x}$$

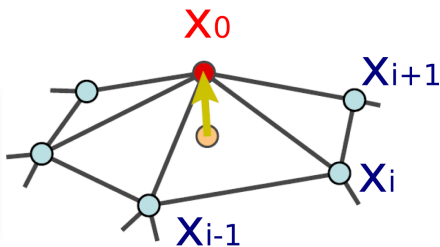
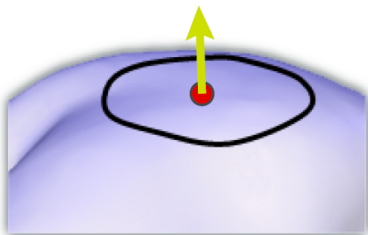
- Comment approximer Δ sur un maillage ?
⇒ Plusieurs solutions, aucune parfaite.

[Wardetzky, Mathur, Kalberer, Grinspun. **Discrete Laplace Operators : No Free Lunch.** *SGP 07*]



- Approximation la plus simple :

$$\Delta \mathbf{x}(\mathbf{x}_0) \simeq \frac{1}{N} \sum_i (\mathbf{x}_i - \mathbf{x}_0) = \bar{\mathbf{x}} - \mathbf{x}_0$$



[Sorkine, Eurographics 05]

- Algorithme du lissage laplacien :

$$\mathbf{x}^{k+1} = \mu \bar{\mathbf{x}} + (1 - \mu) \mathbf{x}^k$$

