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Folded Paper Geometry from 2D Pattern and 3D Contour

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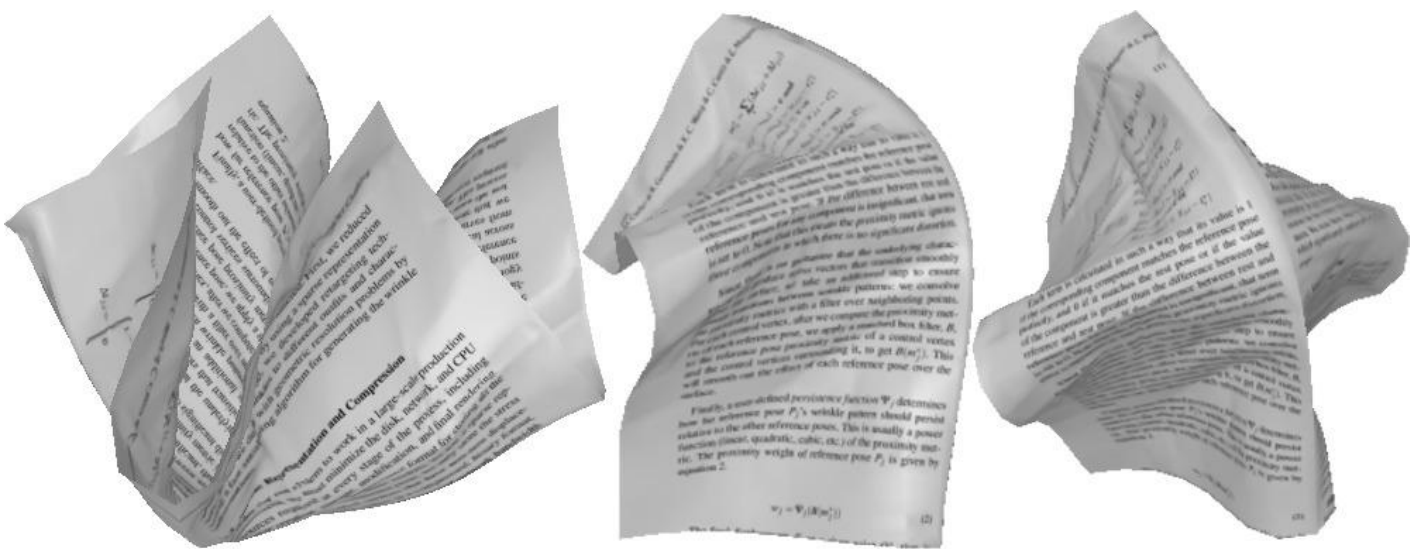
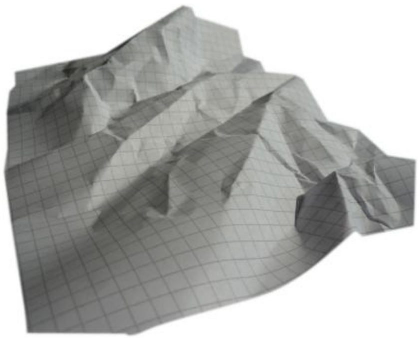


Folded Papers are rare in video-games & CG Movies

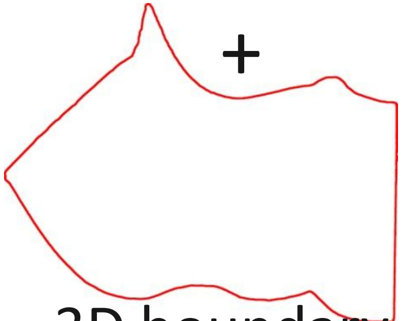


- Few available modeling tools!
  - Non smooth
  - Isometry preserving

- Goal:

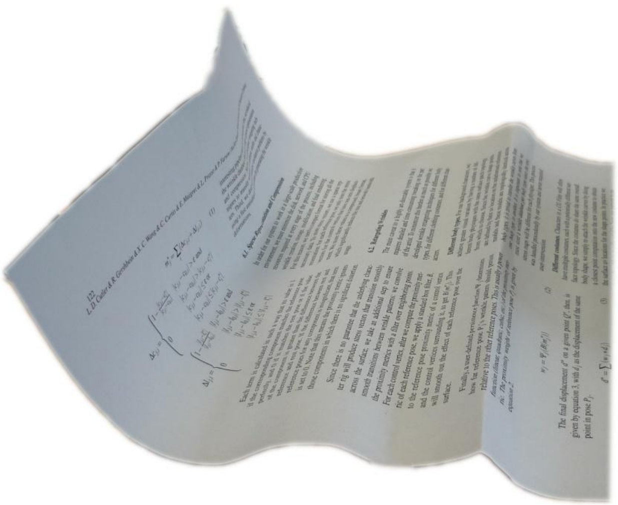
2D pattern

+



3D boundary

→

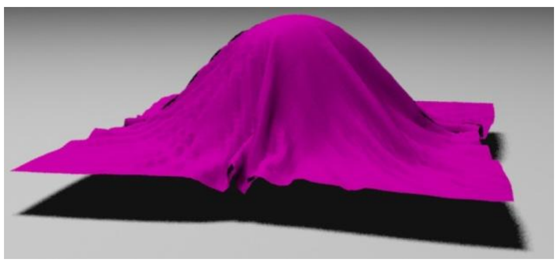


Real pictures

Related Work

- Physically based modeling

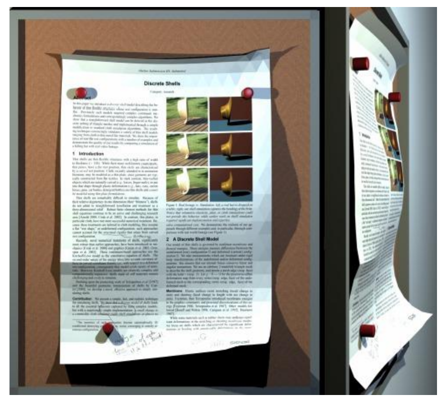
Cloth simulators



[Choi, Ko; TOG 02]
[English, Bridson; TOG 08]
[Thomaszewski et al.; CGF 09]

Slow, Smooth surface

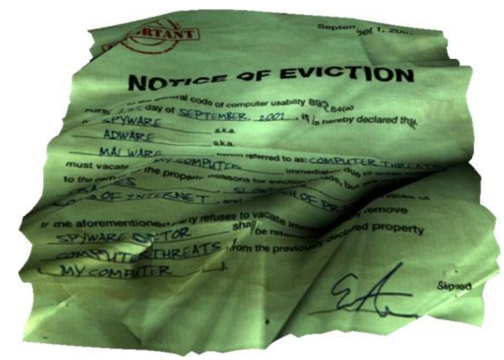
Thin plates from folds



[Burgoon et al.; C&A 06]

Folds are user defined

Specific spring-mass system



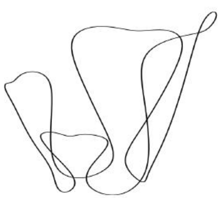
[Kang et al.; CASA 09]

Folds along existing edges

Related Work

- Geometric approaches

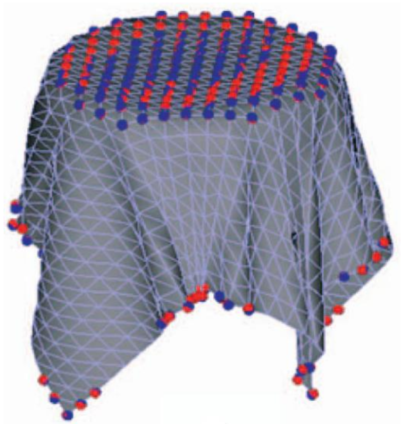
Developable construction



[Frey; CAD 04]
[Rose et al.; SGP 07]

Restricted to the convex hull

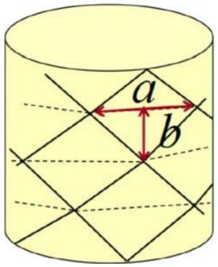
Mesh deformation

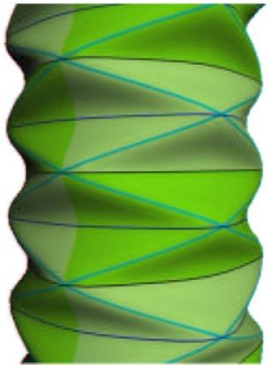


[Tang, Chen; TVCG 09]
[Popa et al.; CGF 09]

Slow, smooth surface

Procedural generation



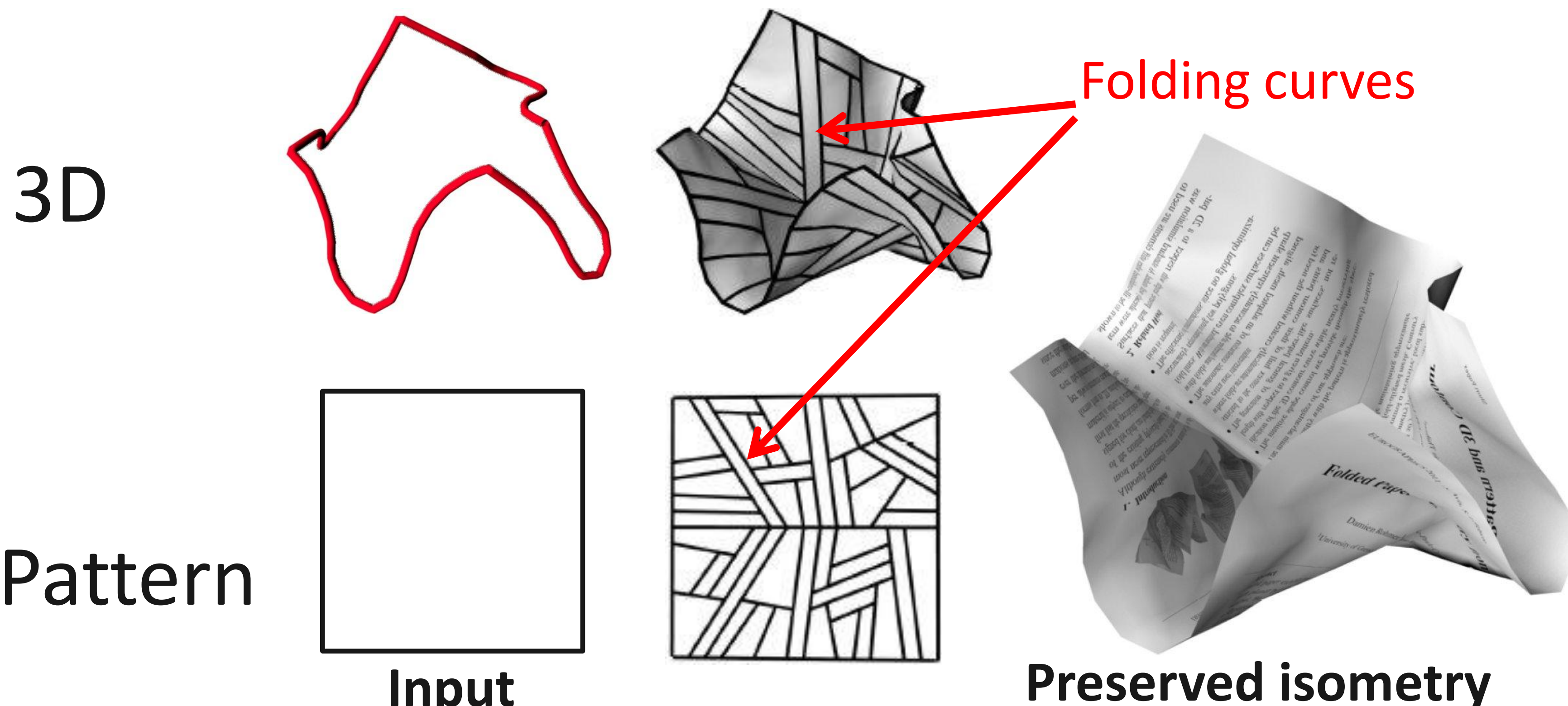


[Decaudin et al.; CGF 06]

Limited range of deformation

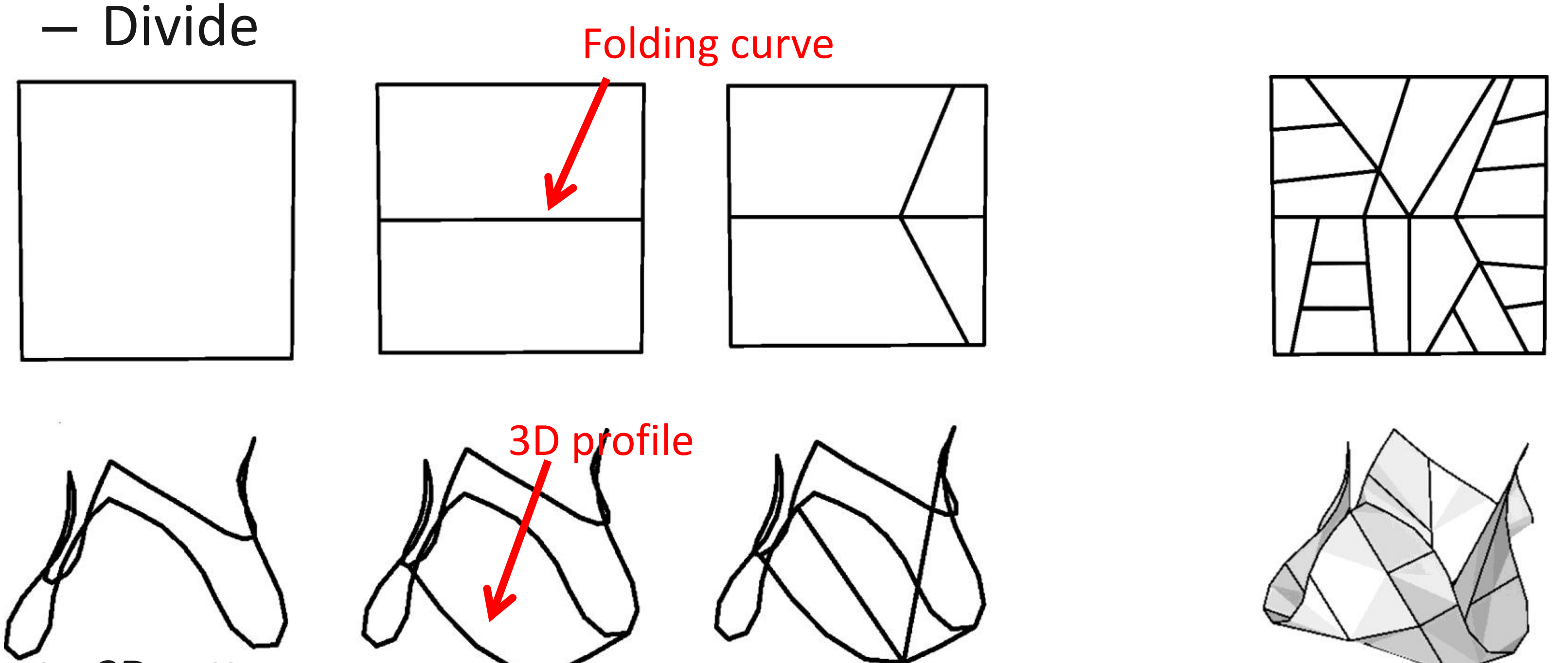
Our Key Idea

- New subdivision improving **length preservation**
- Automatic generation of **folding curves**



Overview

- Divide & Conquer approach
  - **Localize** one fold
  - Compute optimal 3D **profile**
  - Divide



Input = 2D pattern + 3D boundary curve

Subdivision steps …

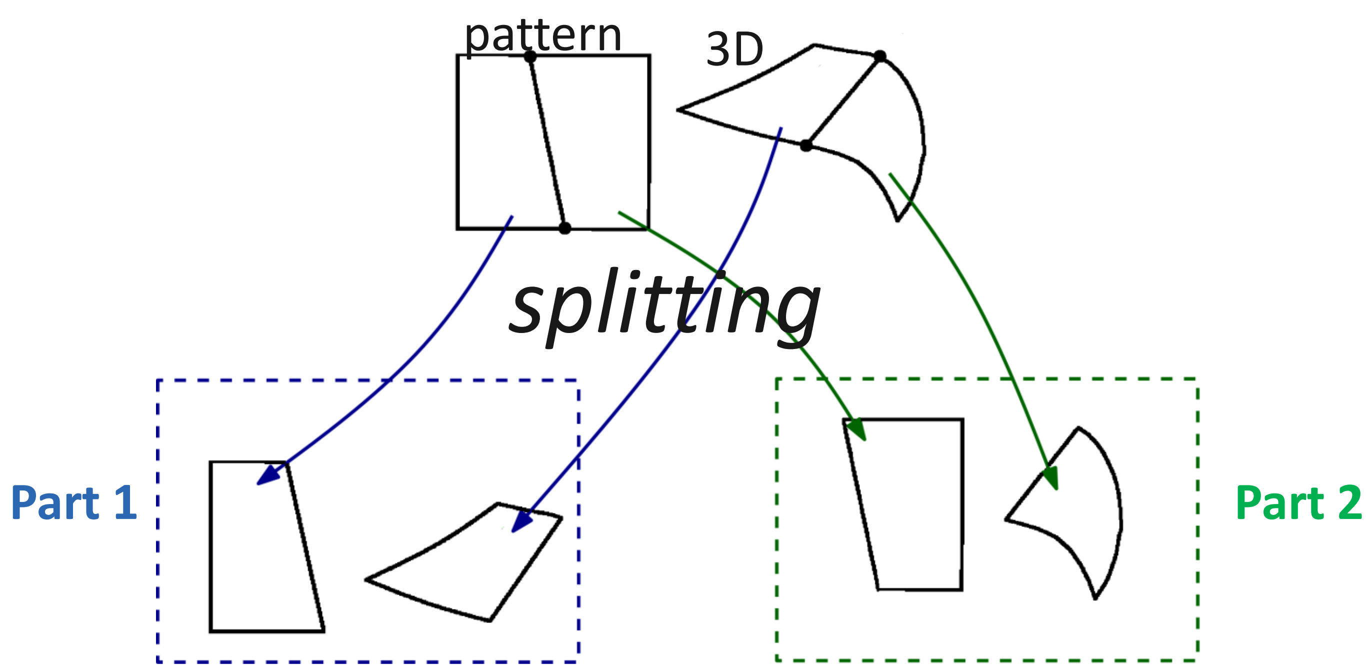
Final folded surface

Recursive subdivision

- **Input**
  - 2D Pattern = convex polygon
  - 3D Boundary = 3D polyline

- **Algorithm**
  1. Localize fold curve
  2. Split into two separated parts
  3. Restart at 1. on the two parts

  Loop until isometry is reached

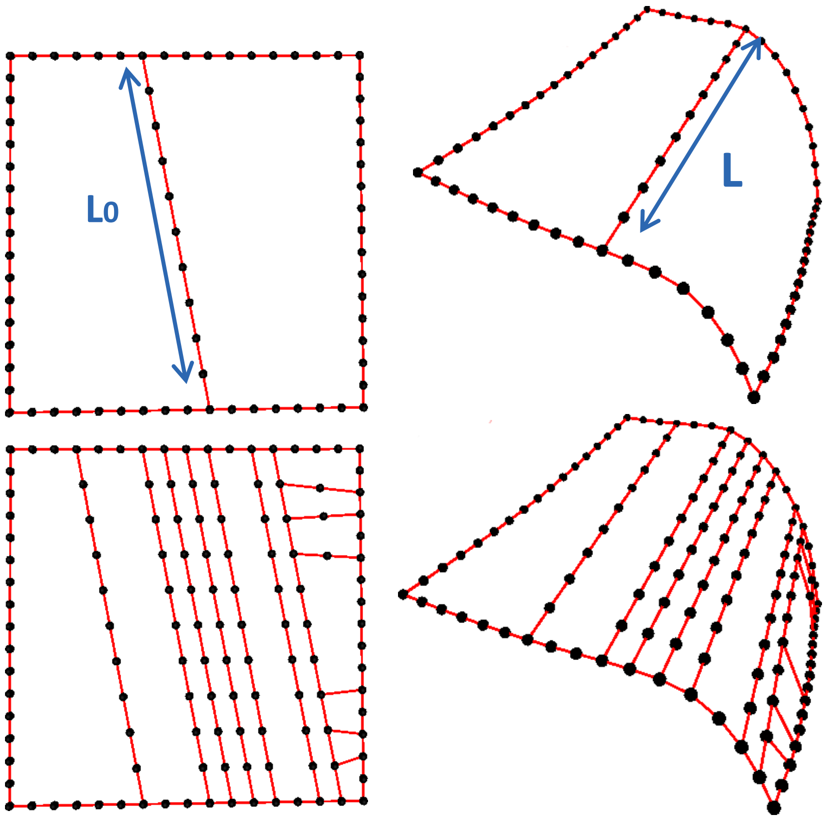


Localizing fold line : straight line

- **Localize = Find *good* pair of vertices**

Case 1: $L=L_0$

=> 2D line mapped in
3D straight line

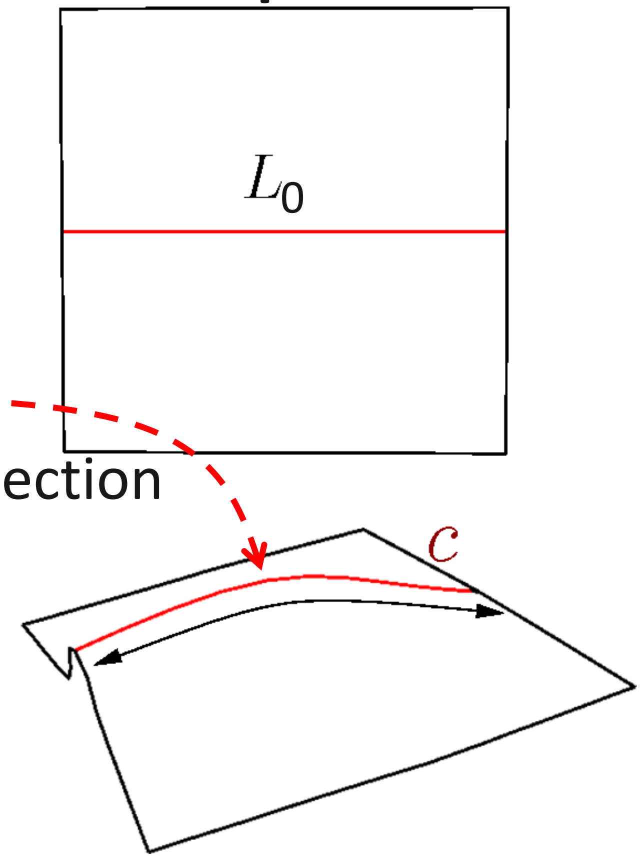


Case 2: $L<L_0$: 3D profile is not a straight line !

Localizing fold line: curved folds

- **Localize = Find pair of vertices with least compression**

Case 2: $L<L_0$



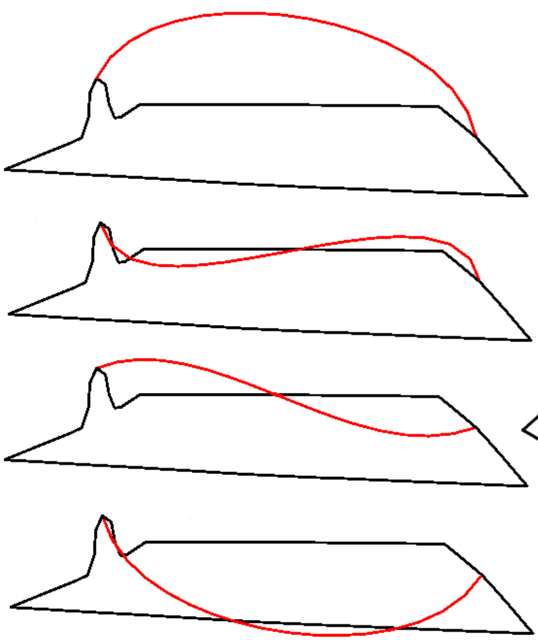
profile = **cubic** polynomial

- **precise**: good approximation of conical section
- **robust**: does not oscillate
- **fast**: limited degrees of freedom

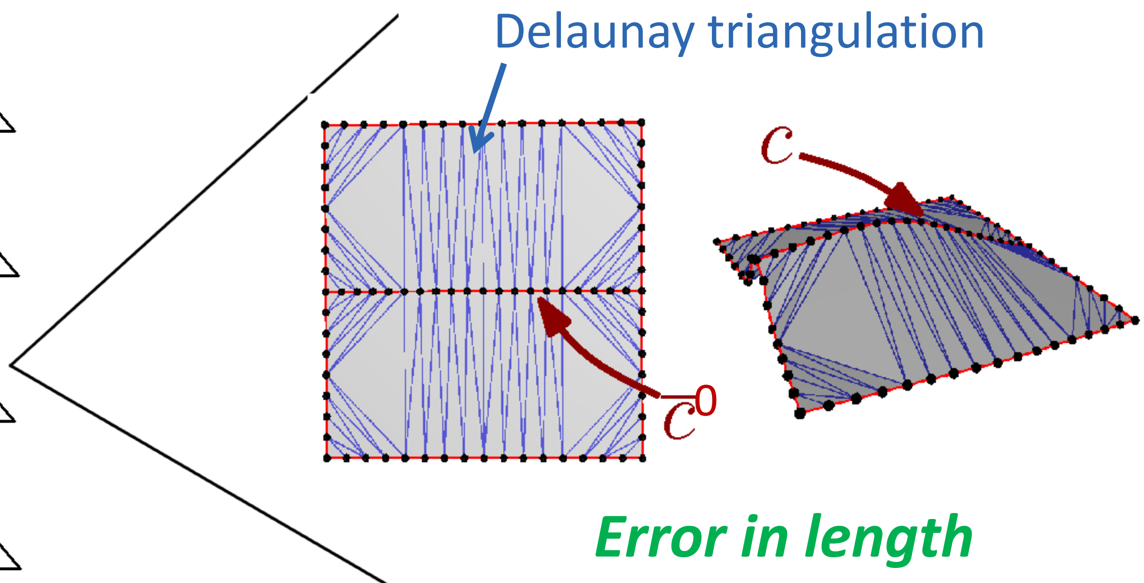
Computing folding profile

Goal: Improve length preservation

=> Find the **best** profile **improving length** preservation



Several possible curves



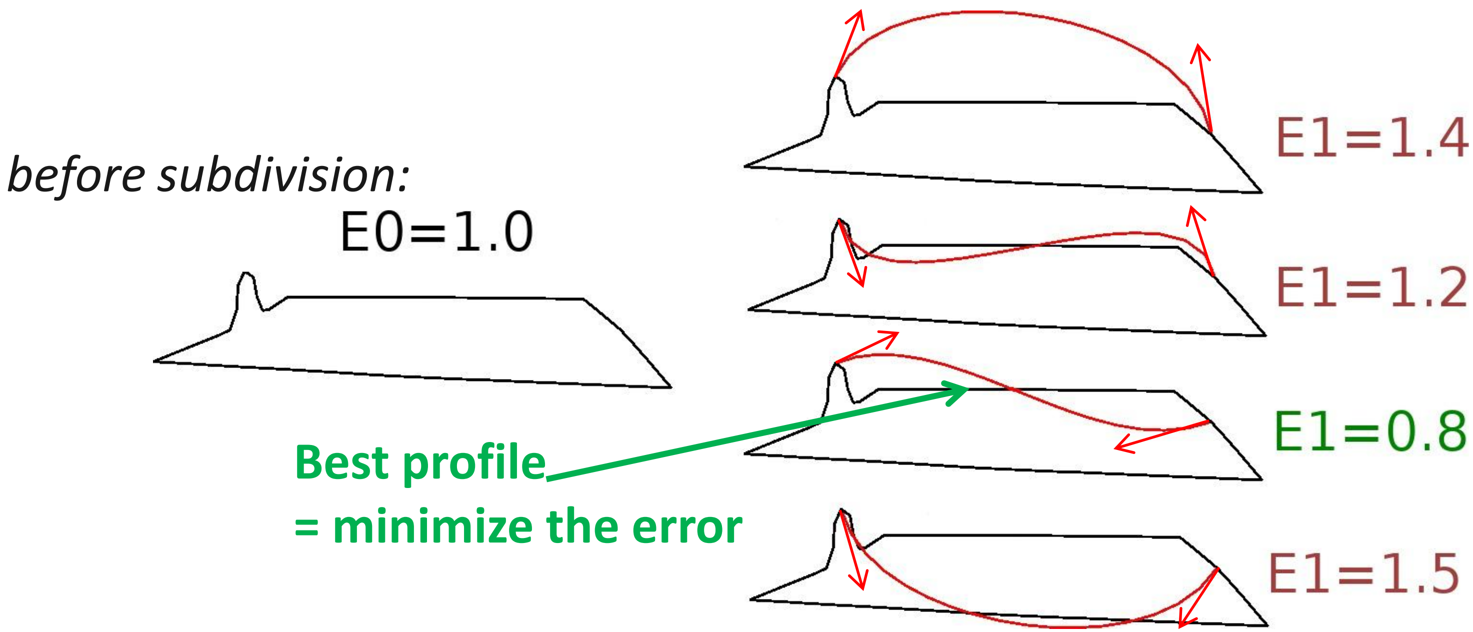
Error in length

$E = \Sigma (L-L0)^2$

Computing folding profile

Goal: Improve length preservation

=> Find the **best** profile **improving length** preservation

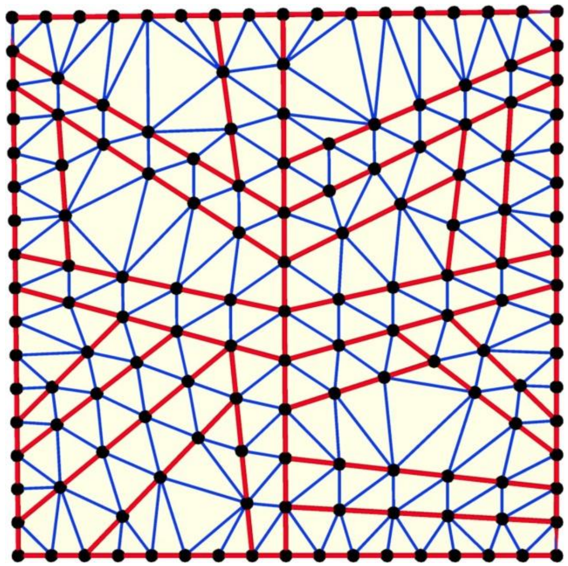


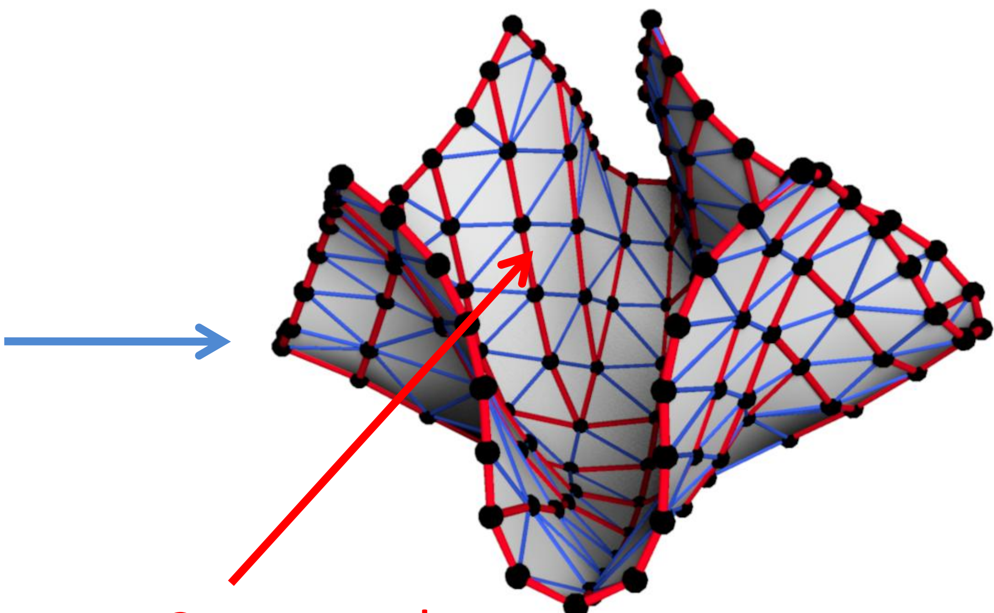
- **Optimization = non linear minimization E=f(curve)**
- **6 degrees of freedom per curve (2 tangents)**
- **Curve is considered if E1<E0**

Final surface

2D Delaunay triangulation

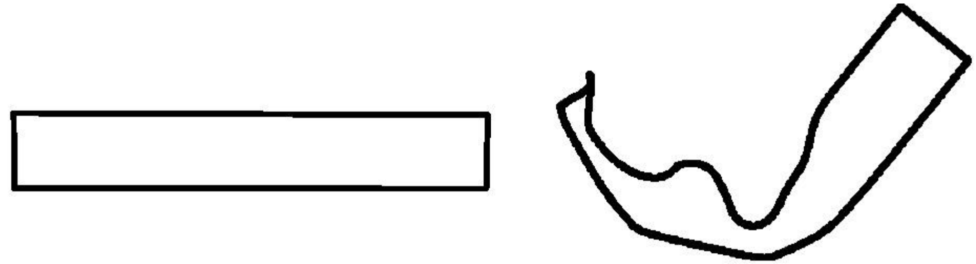
3D mapping



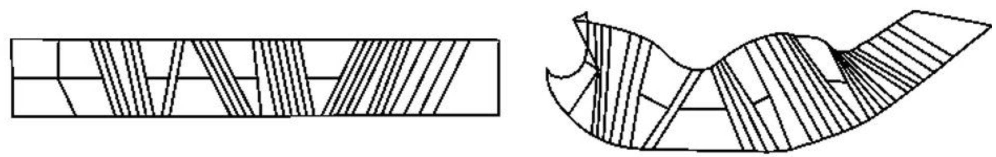


Results : Band strip

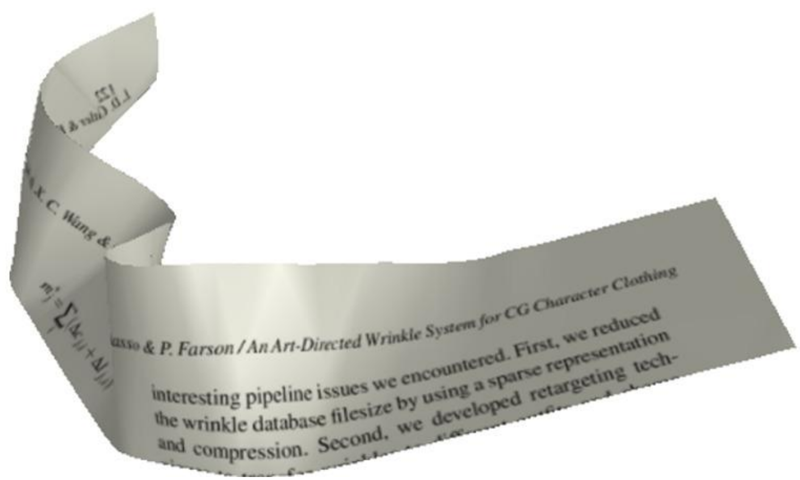
Input



Subdivision

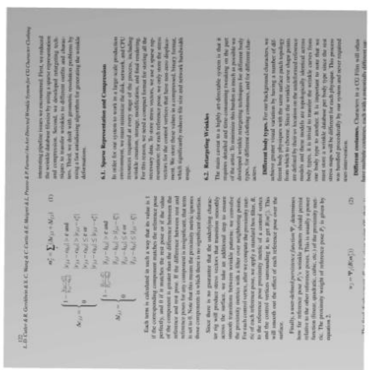


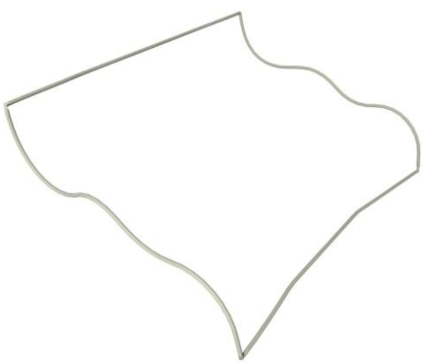
Our textured result



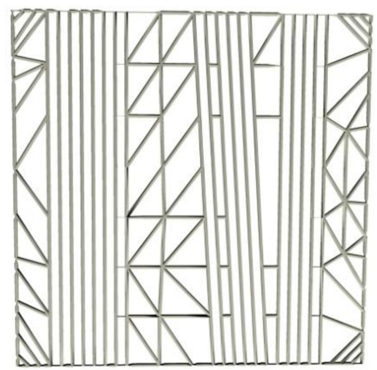
Results : comparison to real sheet

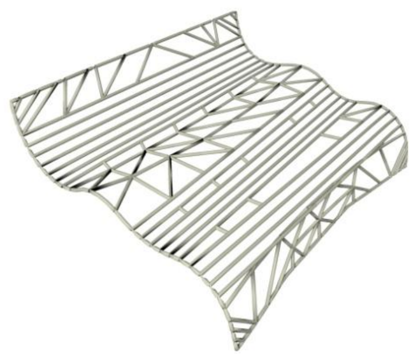
Input





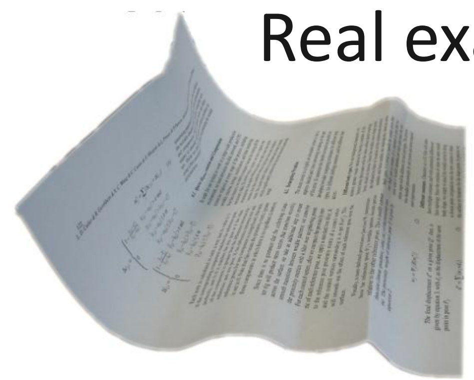
Subdivision



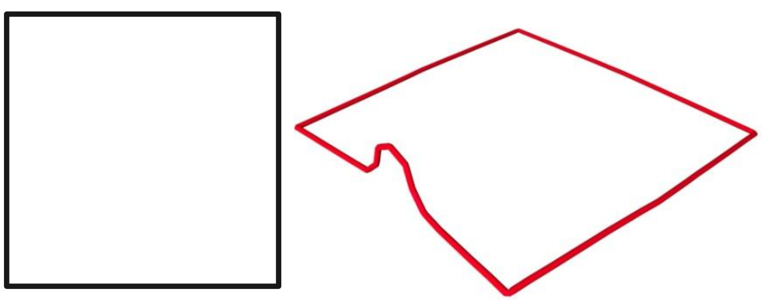


Our result

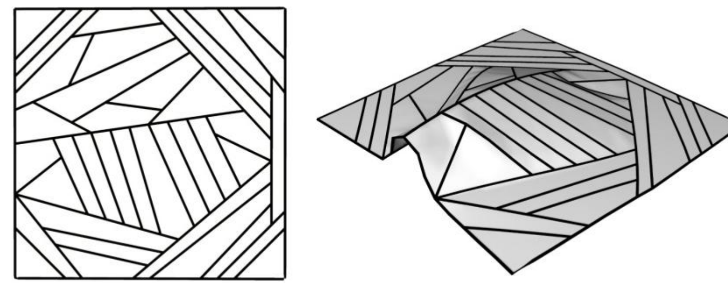
Real example



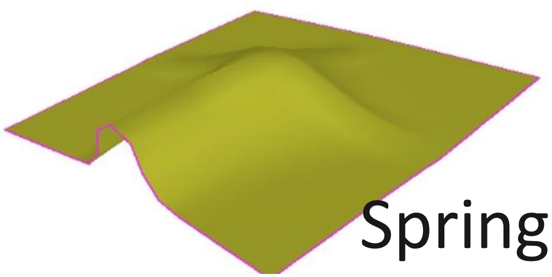
Results : folded paper



Input



Subdivision

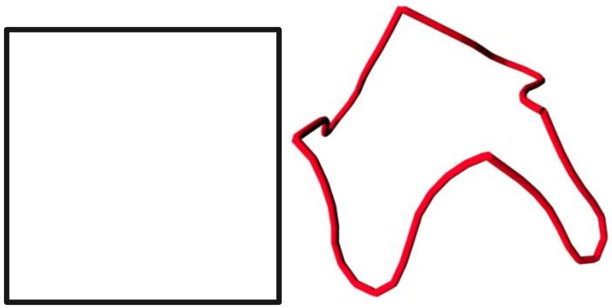


Spring-mass

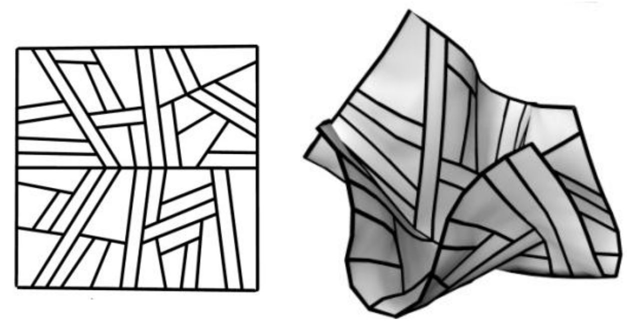
Real paper

Our result

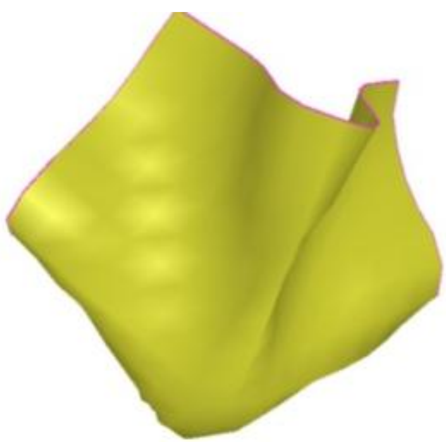
Results: complex folded paper



Input



Subdivision



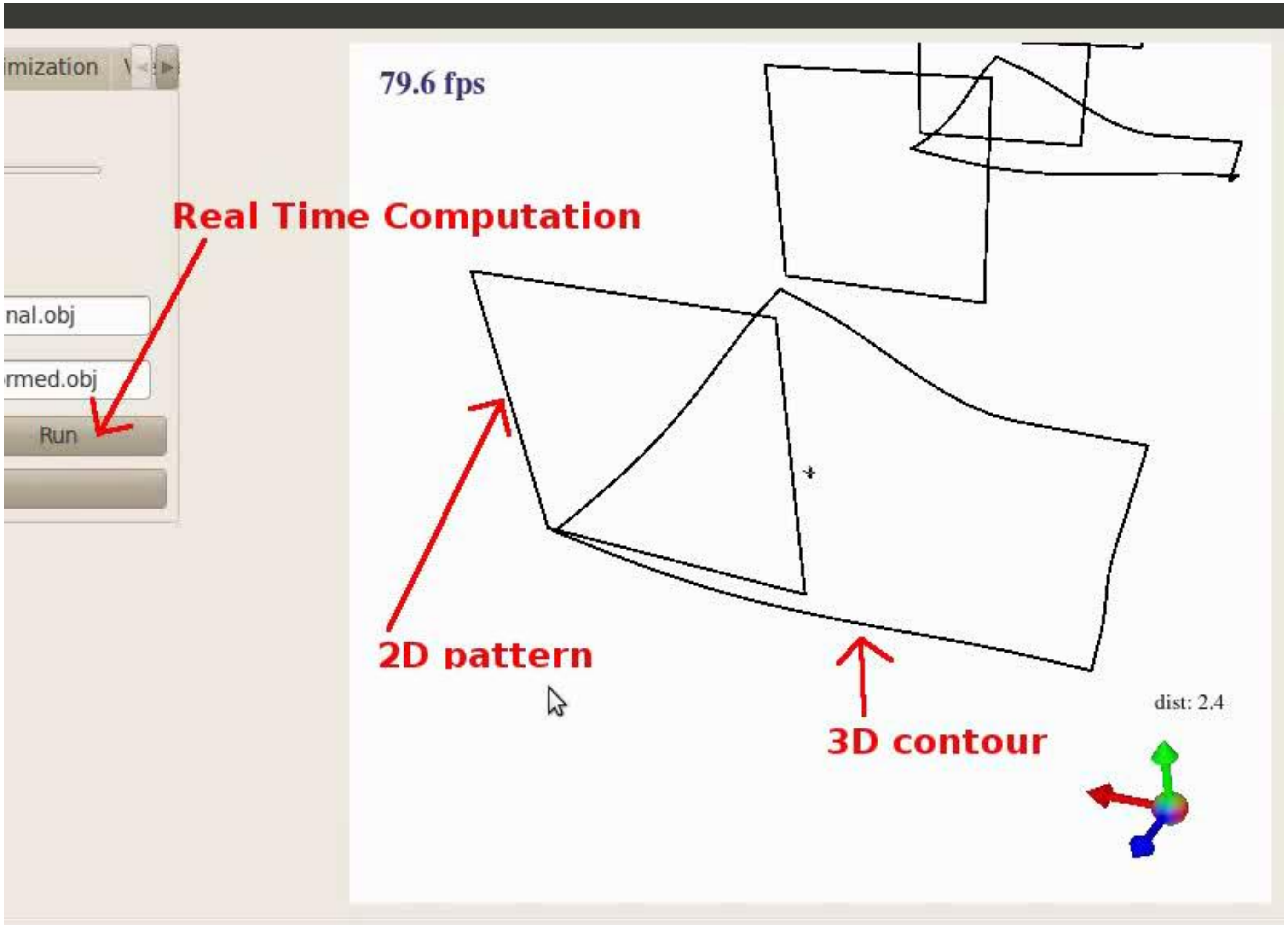
Spring-mass

Real paper

Our result

Results: complex folded paper

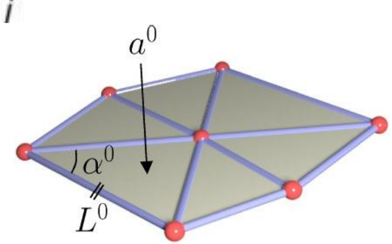
Results: Real time capture

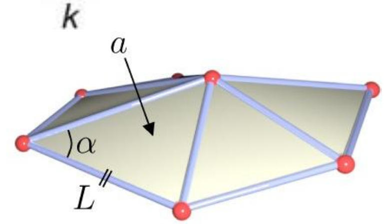


Results: Residual error

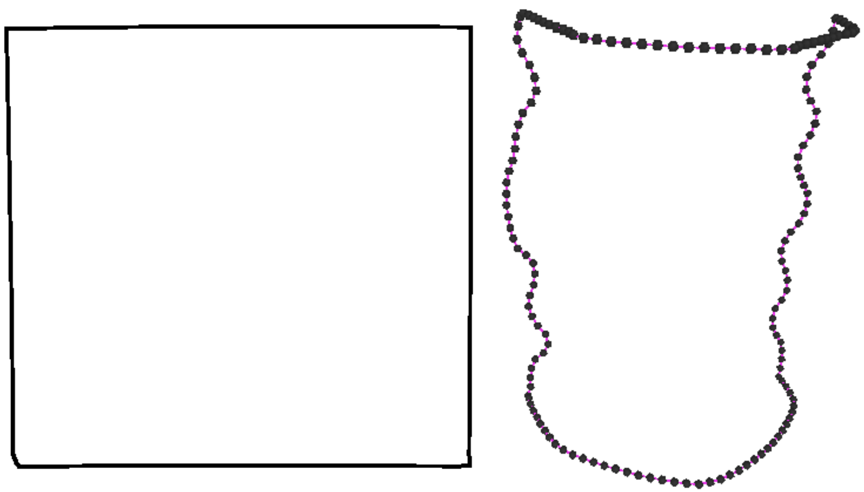
				
Error Length	0.09 (0.21)	0.21 (0.25)	1.12 (2.2)	1.28 (2.5)
Error Angle	0.16 (1.4)	0.35 (1.9)	2.52 (18.3)	2.89 (22.8)
Error Area	0.7 (1.1)	1.3 (1.1)	6.0 (18)	8.0 (18)
Time	<0.1s	<0.1s	0.2s	0.6s

$$E_{\text{length}} = \sum_i (L_i^0 - L_i)^2 \; ; \; E_{\text{angle}} = \sum_i (\alpha_j^0 - \alpha_j)^2 \; ; \; E_{\text{area}} = \sum_k (a_k^0 - a_k)^2$$

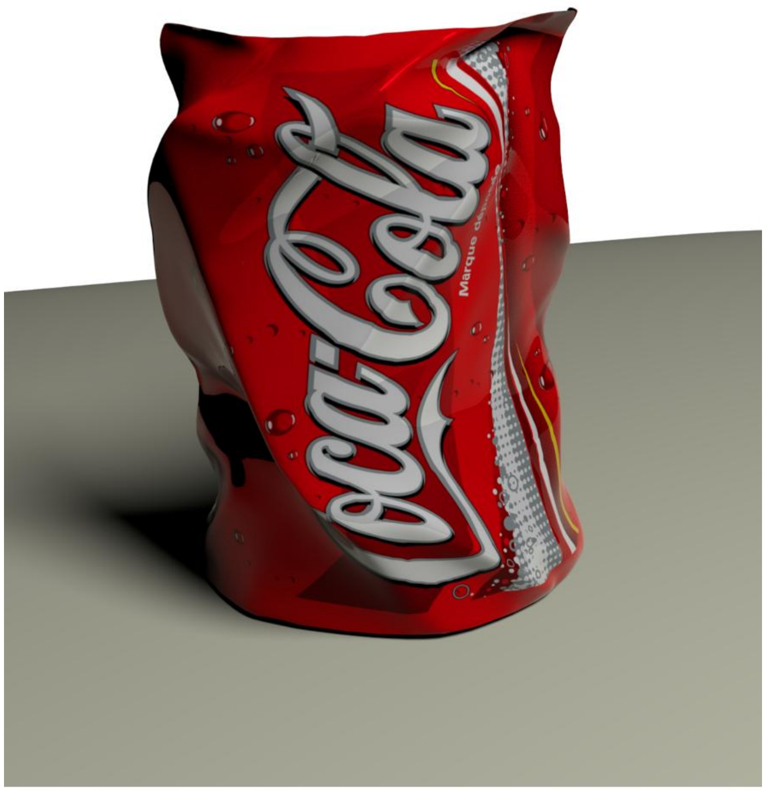




Results: Extension to metal material

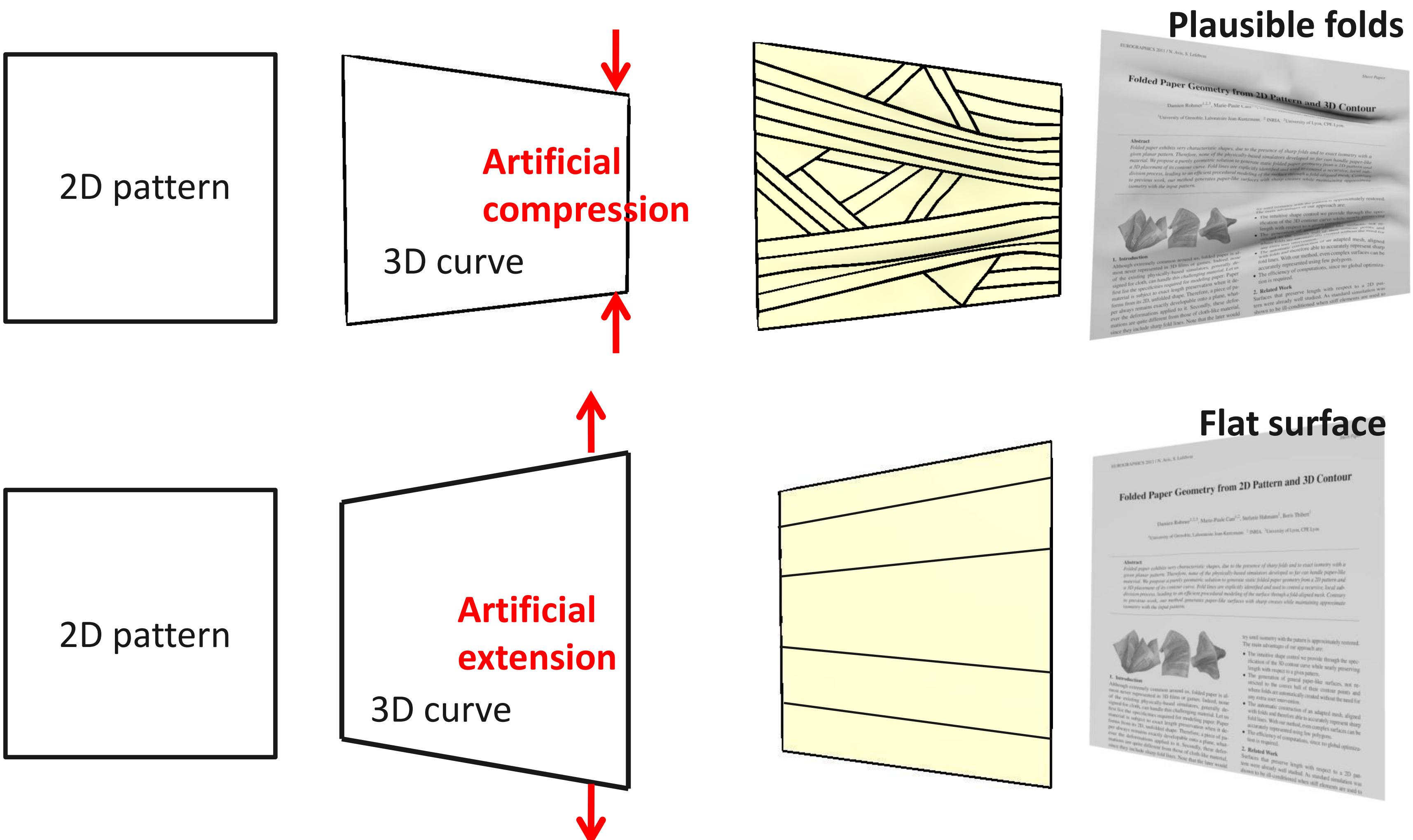


Input



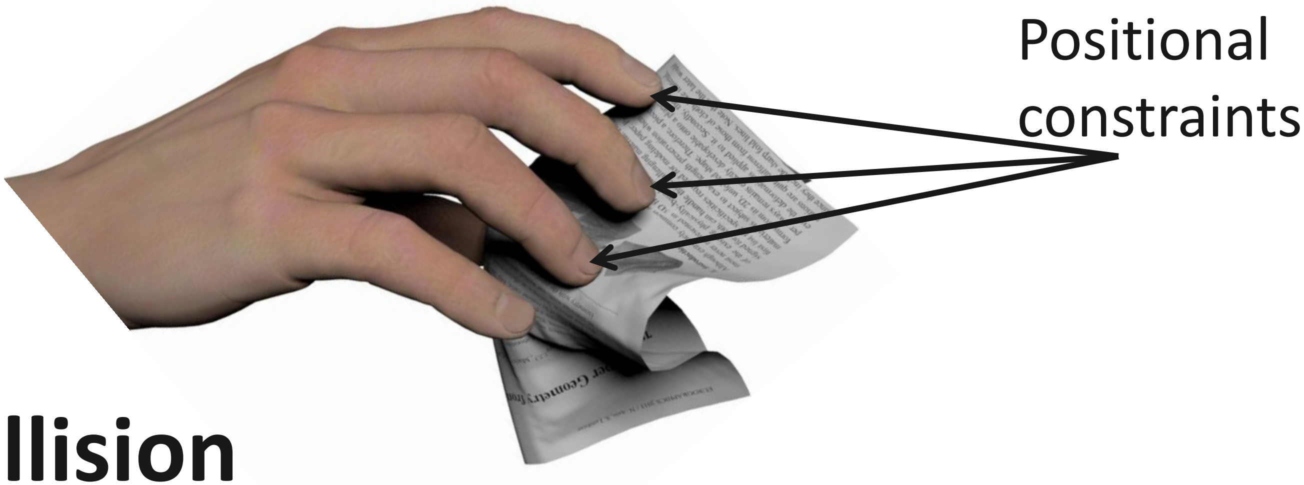
Our result

Results: Robustness to extended/compressed 3D boundary

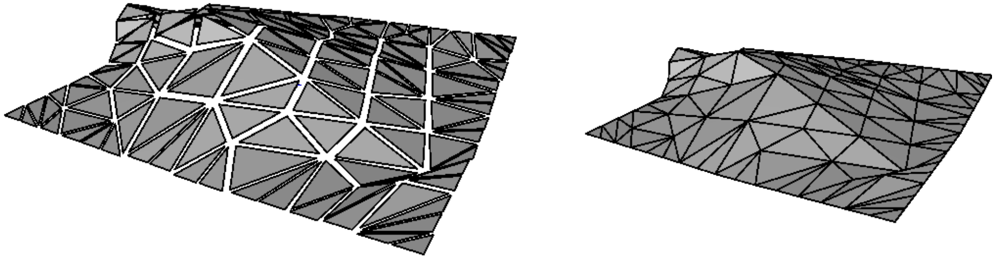


Limitations

- Input



- Self collision
- Error residual



- Static only

Conclusion

New subdivision algorithm

- creates **paper looking** surface
- almost **isometry preserving**

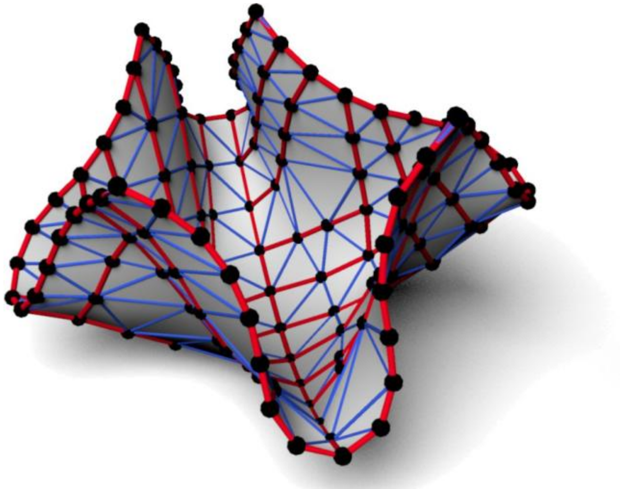
Main ideas:

- **Localize the folds** : *least compression bw vertices*
- **Find the best profile** : *minimizing length error*

+ Fast

+ Non smooth surface

+ Adapted mesh



Thank you